

# Limitations on the Effectiveness of Forward Guidance in the Wake of the COVID-19 Pandemic\*

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Abstract

In the wake of the COVID-19 pandemic, U.S. short-term nominal interest rates are expected to remain at the effective lower bound (ELB) over the next few years, and hence using forward guidance to provide additional monetary stimulus would necessarily involve an even longer time horizon. To shed light on these issues, we consider a canonical New Keynesian model that incorporates realistic features of expectations formation and limitations on the central bank's credibility. In this framework, providing near-term monetary stimulus hinges on making promises of substantial overshooting of output and inflation, and hence forward guidance has tenuous net benefits and may even be counterproductive.

Keywords: Bounded rationality, imperfect credibility, optimal policy

JEL Classifications: E52, E58

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# 1 Introduction

The extraordinary economic crisis precipitated by the COVID-19 pandemic caused historic declines in output and employment that have only been partially reversed since the first wave of the pandemic. The Federal Reserve is now holding the target federal funds rate at the effective lower bound (ELB), and the December 2020 projections of Federal Open Market Committee (FOMC) participants indicated that policy would continue to be constrained by the ELB over the next few years. Consequently, using forward guidance to provide additional monetary stimulus would necessarily involve a much longer time horizon than in the previous experience of the Federal Reserve or any other central bank. Indeed, the Federal Reserve staff concluded in June 2020 that the FOMC "*would have to maintain highly accommodative financial conditions for many years to quicken meaningfully the recovery from the current severe downturn.*"<sup>1</sup>

In this paper, we examine the effectiveness of forward guidance at the effective lower bound (ELB) in the wake of the COVID-19 pandemic. Our analysis builds on key lessons from the recent academic literature about the *forward guidance puzzle*, which highlighted how the efficacy of forward guidance is attenuated by realistic features of expectations formation and limitations on the central bank's credibility in making binding commitments.<sup>2</sup> In fact, recent data underscores the myopia of professional forecasters at the initial stages of the pandemic and the extraordinarily high dispersion of their recent forecasts of U.S. GDP.<sup>3</sup> Moreover, concerns about imperfect credibility have been flagged in past FOMC discussions, reflecting the extent to which the FOMC's composition evolves over time as a result of staggered terms, retirements, and other sources of turnover.

To analyze these issues quantitatively, we formulate a canonical New-Keynesian model which incorporates mechanisms that have been shown to mitigate the forward guidance puzzle, including myopia, imperfect common knowledge, and incomplete financial markets. For each of these specifications, the optimal monetary policy at the ELB is markedly more aggressive than in the conventional model with rational expectations. The initial shortfall of inflation is modest and transitory, while the overshooting of inflation is substantial and

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<sup>1</sup>See FOMC Meeting Minutes (2020), p. 3. Reifschneider and Williams (2000) analyzed the policy implications of the ELB using the FRB/US staff model; see also Reifschneider and Roberts (2006), Williams (2009), and Brayton, Laubach, and Reifschneider (2014). More recently, Bernanke, Kiley and Roberts (2019), Reifschneider and Wilcox (2019), Hebden et. al. (2020), and Arias et. al. (2020) examined these issues in a variant of the FRB/US model where investors have rational expectations while other private agents have adaptive expectations.

<sup>2</sup>Del Negro, Giannoni and Patterson (2012) made a seminal contribution in identifying the forward guidance puzzle. See also Kiley (2016).

<sup>3</sup>Altig et al (2020) and Coibion, Gorodnichenko and Weber (2020) also document a large degree of uncertainty during the Covid-19 crisis; see also Bordo, Levin and Levy (2020).

persistent.

Next, we incorporate limitations on the central bank's credibility in making longer horizon commitments regarding the path of monetary policy. In particular, the central bank announces its commitment in the initial period when policy becomes constrained by the ELB, but private agents perceive a fixed probability that in any given period the central bank may renege on that commitment and revert to the purely discretionary policy. This form of imperfect credibility induces a marked deterioration in macroeconomic stability, regardless of the particular specification of expectations formation.

We also consider the implications of model uncertainty, i.e., policymakers do not have full knowledge of the dynamic structure of the economy. To illustrate this issue, we consider scenarios in which the central bank does not observe the actual slope of the New Keynesian Phillips curve. If the central bank incorrectly formulates its strategy based on a steeper slope, the misperception results in a dramatic set of policy errors due to the mistaken strong feedback loop between output gap, the inflation rate, and the real interest rate.

Our framework makes a novel contribution in nesting several distinct mechanisms of expectations formation into a single NK model, including incomplete credit markets, imperfect common knowledge, and bounded rationality; see McKay, Nakamura and Steinsson (2016), Angeletos and Lian (2018) and Gabaix (2020) respectively. This nested representation facilitates quantitative comparisons regarding the implications of these alternative mechanisms in mitigating the efficacy of forward guidance relative to the baseline specification of rational expectations. For example, myopic expectations yield a more substantial degree of inflation overshooting than either imperfect common knowledge or incomplete markets.

In characterizing the implications of these mechanisms for the design of optimal monetary policy at the ELB, we draw on numerous prior studies that have investigated these issues in models with rational expectations and complete risk sharing.<sup>4</sup> Our work is also closely related to Gabaix (2020) and Benchimol and Bounader (2020), who have characterized optimal policies under bounded rationality when monetary policy is not constrained by the ELB.<sup>5</sup>

Our approach to analyzing imperfect credibility is distinct from the analysis of "loose commitments" at the ELB; see Bodenstein, Hebden, and Nunes (2012).<sup>6</sup> That approach

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<sup>4</sup>Jung, Teranishi and Watanabe (2001, 2005), Eggertsson and Woodford (2003), Adam and Billi (2006), Nakov (2008), and Levin, Lopez-Salido, Nelson and Yun (2010) analyze optimal monetary policy at the ELB in NK models with rational expectations.

<sup>5</sup>Gabaix (2020) also analyzed the joint design of monetary and fiscal policies at the ELB.

<sup>6</sup>In the stochastic planning or loose-commitment approach of Roberds (1987) and Schaumburg and Tambalotti (2007), the policymaker is able to commit to future policies, but each period there is an exogenous probability a new policymaker will be appointed. Since future policymakers are not bound to keep the promises of their predecessors, the current promises are discounted by the probability that the policymaker in office will be replaced. In the related approach of Debortoli and Nunes (2010), and Debortoli, Maih and Nunes (2014), there is a fixed exogenous probability each period that the policymaker reoptimizes. Since the

assumes that in any given period the central bank may renege on its prior plan and announce a new plan, and hence private agents discount the likelihood that the central bank will carry through with any particular announcement. By contrast, our framework assumes that the central bank follows the optimal policy under discretion if it reneges on the commitment policy conveyed in its initial forward guidance, consistent with the notion that any subsequent announcements might well be dismissed by private agents once the central bank has reneged on a previous promise.

Our analysis is also distinct from previous studies that have investigated the design of sustainable monetary policy plans.<sup>7</sup> In particular, the sustainability of forward guidance at the ELB hinges on the duration and frequency of such episodes; see Walsh (2018) and Nakata and Sunakawa (2019). That approach is particularly relevant for analyzing policies for mitigating routine economic shocks, whereas our framework seems appropriate for analyzing forward guidance in the context of a rare event like the COVID-19 pandemic.

Overall, our analysis highlights the fundamental challenges that monetary policymakers may face in fostering economic recovery and price stability in the wake of the COVID-19 pandemic. In particular, our results indicate that expectations formation and imperfect credibility substantially limit the efficacy of forward guidance, especially over longer time horizons. In such circumstances, central banks may need to rely even more heavily on deploying balance sheet tools, which are also subject to potential limits on efficacy as well as significant costs and risks.<sup>8</sup> These considerations underscore the rationale for central banks to move forward with changes in the payment system that would eliminate the ELB as a constraint on monetary policy.<sup>9</sup>

The remainder of this paper is organized as follows: Section 2 presents evidence on myopia and dispersion in expectations formation. Section 3 presents the small-scale New Keynesian model, and 4 discusses the optimal policy characterization in this model. Imperfect credibility is introduced in section 5 and preliminary aspects of model uncertainty are discussed in section 6. Section 7 concludes.

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policymaker may reoptimize in the future and does not follow through on previous policies, current promises are discounted.

<sup>7</sup>Both analyses also model the contractionary shock and as a two-state Markov process. In the sustainable plans approach of Chari and Kehoe (1990), which is also used by Walsh (2018) and Nakata and Sunakawa (2019), any commitment technology is absent. The policymaker honors a previous promise only if doing so is the best strategy for her at the time the promise needs to be honored.

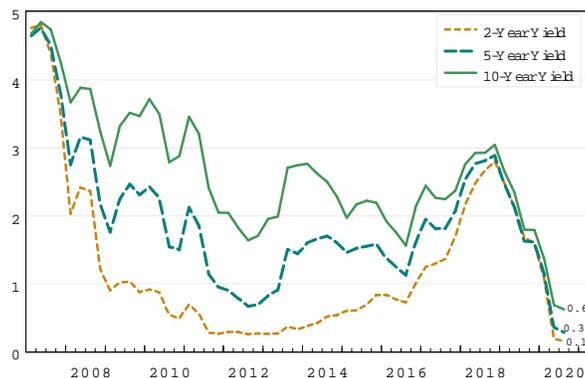
<sup>8</sup>Issues with the use of balance sheet tools are also explored in Greenlaw, Hamilton, Harris and West (2018) and Bordo and Sinha (2020).

<sup>9</sup>These issues are discussed in Bordo and Levin (2017) and Bordo and Levin (2019).

## 2 Evidence on Expectations about the Economy and Policy

The COVID-19 pandemic has led financial market participants to expect a very slow economic recovery. The nominal term structure of interest rates illustrates this in figure 1. The 10-year yield is far lower in July 2020 than at any point during the 2008 financial crisis; the 2- and 5-year yields have fallen very sharply in the last quarter.

Figure 1: Term Structure of Yields



Note: These are the quarterly averages of the 2-, 5- and 10-year nominal yields from St. Louis Fred database. The yields are shown in percentages.

We seek to get a better understanding of how expectations of investors have responded in the run-up to the unprecedented COVID-19 crisis. To do this, we examine the Federal Reserve Bank of Philadelphia’s Survey of Professional Forecasters (SPF) and Survey of Primary Dealers (SPD) conducted by the Federal Reserve Bank of New York. The SPF has been administered by the Philadelphia Federal Reserve since 1990, and participants include financial market professionals, academics and other researchers. The survey is conducted on a quarterly basis. The SPD has been conducted by the Federal Reserve Bank of New York since 2011.<sup>10</sup> The respondents belong to primary dealer firms, and the survey questions are designed to ascertain expectations about these firms. The survey questions are circulated among the respondent groups two weeks before each FOMC meeting, and the results are made available on the website approximately three weeks after the FOMC meeting.

<sup>10</sup>The survey is on the Federal Reserve Bank of New York’s website: [https://www.newyorkfed.org/markets/primarydealer\\_survey\\_questions.html](https://www.newyorkfed.org/markets/primarydealer_survey_questions.html)

We use two aspects of the surveys: first, we use the SPF to examine the myopia and disagreement about the evolution of the real GDP and unemployment during the 2020 crisis<sup>11</sup>. We also use the SPF expectations from the 2008 crisis to explore these characteristics about investor expectations further. As our model below shows, both myopia and disagreement would impede the effectiveness of forward guidance policy measures. Second, we examine the SPD from 2011 and 2012 to understand whether, and by how much, financial market participants were changing their expectations about a liftoff in the Federal Funds rate in response to forward guidance from the FOMC. As the ELB was reached in December 2008, the FOMC began to offer different variants of forward guidance at various meetings. We examine the effects of the forward guidance statements over the ELB period examining the expectations of liftoff reported by the primary dealers in the Federal Reserve Bank of New York survey, as well as by using changes in nominal and real yields.

We also examine transcripts of the FOMC meetings in 2011. These discussions of FOMC members give unique insights into the concerns raised about the impact of market expectations on the effectiveness of forward guidance on financial market participants. There are also insightful discussions related to the formulation of forward guidance over extended periods.

## 2.1 Expectations about the Economy

Myopia in expectations reported by the SPF participants is evident for the 2020 as well as the 2008 crises. The forecasters are asked to assign probabilities to the annual-average over annual-average percent change in the level of real chainweighted GDP. These are shown in figure 2. We consider surveys in the quarters preceding, and at the start of, the crises of 2008 and 2020.

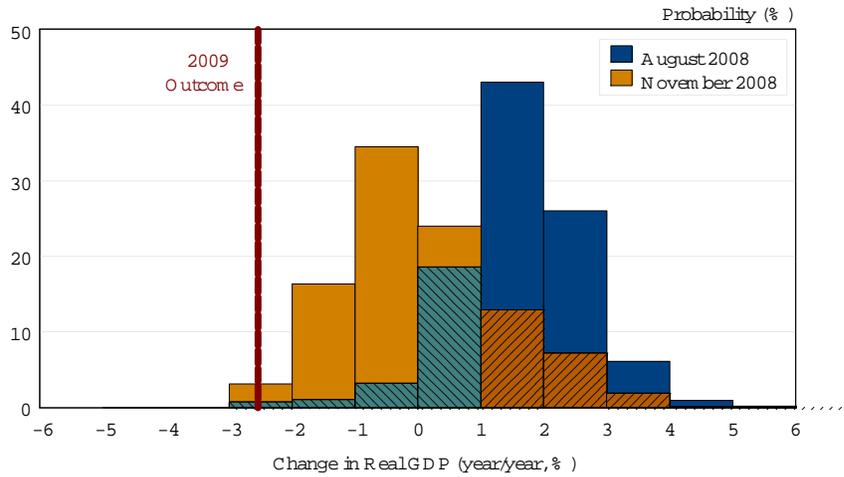
The first panel of figure 2 shows the results for the 2008 crisis. The actual annual-average over the annual-average percent change in the real GDP is indicated by the red line. In the August 2008 SPF, forecasters placed insignificant probabilities of a decline in this growth rate.<sup>12</sup> By the next survey, in November 2008, the mass of probabilities was still concentrated at significantly higher growth rates than the actual outcome.

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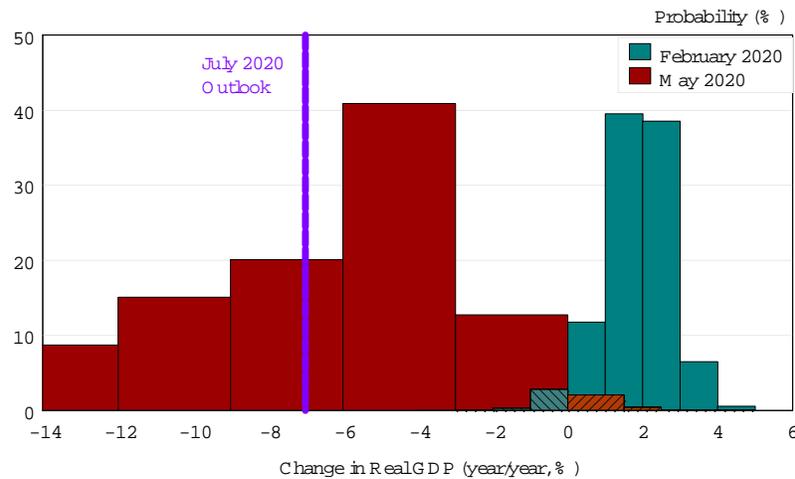
<sup>11</sup>We also examine the 2020 SPD expectations for myopia and disagreement. The results are noted in appendix A, and corroborate the findings from the SPF.

<sup>12</sup>Among the individual forecasters, only one of the 43 forecasters placed any probability of the decline in the growth rate.

Figure 2: Myopia among SPF Forecasters  
 Panel A: 2007-09 Crisis



Panel B: 2020 Crisis

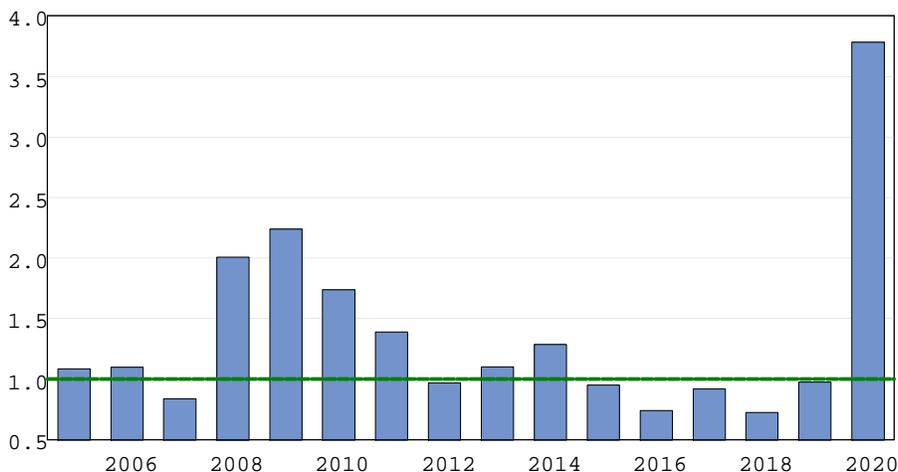


Note: These are the probabilities assigned by the SPF forecasters to the annual-average over annual-average percent change in the level of real chainweighted GDP.

Next, we consider myopia in expectations during the 2020 COVID-19 pandemic. In panel B of figure 2, the July 2020 outlook is from the forecasts of the Conference Board. In the February 2020 survey, forecasters were very optimistic about the economy; by May 2020, the mass of the distribution had shifted to a much more pessimistic view.

Dispersion of forecasts among the SPF participants has been historically unprecedented in the 2020 crisis, and this is illustrated in figure 3. We present the Interquartile Ranges (IQR) of the one-year ahead forecasts for the log of the real GDP from 2005-2020. The non-recession means for the IQRs are also indicated by the green lines. While dispersion doubled during the 2008 crisis, the 2020 crisis recorded the highest dispersion on record.

Figure 3: Dispersion among SPF Forecasters



Note: These are the annual averages of the Interquartile Ranges (IQRs) or D1 measure of the 4-quarter-ahead forecasts for  $\log(\text{real GDP})$  reported by the forecasters in the Federal Reserve Bank of Philadelphia’s Survey of Professional Forecasters. The non-recession mean is noted by the green line.

These findings suggest that myopia and forecast dispersion are important characteristics of forecaster beliefs. Since these are professional forecasters, and their expectations are considered to be a gauge for expectations more generally, these are important features to incorporate in an exploration of the effectiveness of forward guidance.

## 2.2 Expectations about Forward Guidance

We now turn to examining the evidence from the SPD about how expectations changed in response to the issuance of specific forward guidance. We then analyze the FOMC transcripts. We are considering the responses of forecasters to forward guidance issued by the Federal

Reserve during and in the aftermath of the 2008 crisis. In December 2008 and March 2009, the statements communicated that the rates would remain low for "some time" or an "extended period". In August 2011 and January and September 2012, this was replaced by calendar based guidance. Table 1 below shows the highlights of these statements.

**Table 1: Dates and Highlights of Forward Guidance**

<i>Date</i>	<i>Forward Guidance Highlights</i>
Dec 16, 2008	"...exceptionally low levels of the federal funds rate for some time."
Mar 18, 2009	replaces "for some time" with "for an extended period"
Aug 9, 2011	federal funds rate at exceptionally low levels "at least through mid-2013."
Jan 25, 2012	replaces "at least through mid-2013" with "at least through late 2014."
Sep 13, 2012	"at least through mid-2015."

Note: These are the dates on which the FOMC offered forward guidance. The full list of dates is available at <https://www.federalreserve.gov/monetarypolicy/timeline-forward-guidance-about-the-federal-funds-rate.htm>. Early stages of FG simply interpreted as 6 months; in 2011 and 2012 introduced calendar based guidance for 2-21/2 years; outcome based guidance was not interpreted as changing liftoff median liftoff dates by more than 2 quarters. Also put term structure of yields across the two periods.

### 2.2.1 Survey Evidence

The responses from the SPD suggest that in response to the forward guidance statements there were only small changes in the expectations of forecasters regarding the timing of liftoff in the federal funds rate. We use the following question asked by the SPD: "*Of the possible outcomes below, please indicate the percent chance you attach to the timing of the first federal funds target rate increase.*" The question makes available time periods, and the respondents are required to assign probabilities of liftoff to the different time periods. The format of a sample question, along with the responses, is shown in table A1 of appendix A.

Using this question, we construct a series of the median liftoff date reported by the respondents. The results indicate that the primary dealers shifted their expectations about the liftoff dates by a maximum of two quarters between 2011 and 2012. Thus, while the financial market participants were responding to the forward guidance provided, the effects were quite short-term. This change in the liftoff dates are shown in table A2 of appendix A.

We also examine the effects of the forward guidance announcements on the nominal and real term structure of yields. These findings confirm those in the literature: the largest change, across the term structure for both nominal and real yields occurred around the March 18, 2009 announcement. On this day, in its post-meeting statement, the FOMC replaced the phrase "for some time" with "for an extended period" in reference to the ELB range for the Federal Funds Rate. By the time the 2012 forward guidance was being offered, its effects on yield changes were much smaller. The patterns in the yield changes suggest that the first instances of forward guidance had large effects on the term structure of yields, but calendar-based guidance was less effective. These responses are shown in table A3 of appendix A.

These findings from the survey expectations, as well as the yield changes, suggest that the forward guidance offered by the FOMC is able to affect financial market expectations at relatively short-term horizons.

### **2.2.2 Discussion of Forward Guidance in FOMC Transcripts**

The FOMC excerpts highlight the broad concerns of policymakers in using forward guidance, and of making commitments at long-term horizons. A common characteristic of the forward guidance offered in 2011 and 2012 was that it referred to horizons of approximately two years. For example, at the August 9, 2011 meeting, the forward guidance only extended upto "...at least through mid-2013". However, as a protracted ELB episode would require forward guidance to be offered for a time period far in the future, it is instructive to examine the transcripts of the FOMC during the time the forward guidance discussions were taking place. During the FOMC meeting held on August 9, 2011, there were several concerns raised by the committee members about the nature and period of the forward guidance which would be offered. For example, President Eric Rosengren, of the Federal Reserve Bank of Boston, stated:

I fear that the change in language we have discussed does little to signal further easing. It says that exceptionally low levels of the federal funds rate are likely to be maintained at least through mid-2013, but this matches the path of the federal funds rate that the market already expects [...]. A statement that matches current market expectations is not likely to have much of an effect on market rates.

Further discussions also noted the contingent nature of the forward guidance offered. Federal Reserve Bank of New York President, and the FOMC Vice-President Bill Dudley noted:

“...making binding commitments might be viewed as potentially reckless in a world where the outlook is highly uncertain.”

Likewise, Janet Yellen (then serving as Federal Reserve Board vice chair) indicated:

“...we need to be mindful of the intrinsic limits on our ability to make credible promises over time horizons that extend beyond several years. We need to follow a pragmatic approach for promoting the stability of economic activity and inflation, recognizing the limits of our understanding of the structure and evolution of the economy and of our ability to anticipate or plan for all possible contingencies.”

During the November 1-2, 2011 meeting, the FOMC participants expressed concern about how the private sector would view the forward guidance being offered, in the case of Committee decisions that were not unanimous. Governor Elizabeth Duke stated:

If we’re going to convince the public that we’re going to act differently in the future, especially far into the future, we will first need unanimous or near unanimous agreement and steadfast conviction that it’s the right course for us to follow [...]. Without near unanimity and one voice, the public could focus on the potential for the rotation of voters to change the path or the potential for the two open seats and the upcoming term endings on the Board to bring about a philosophical change or, in the worst case for credibility, the political debate could become fixated on effecting such a change through legislation or personnel changes.

We now turn our attention to modeling these concerns in a stylized NK framework.

### 3 Model Framework

Here we specify the model framework which we use to characterize the findings on private sector expectations. The formulation nests three of the prominent models that were originally put forth to address the forward guidance puzzle.<sup>13</sup> However, each one presents a plausible way to model expectations as we observe above. Our objective is to characterize and analyze the optimal policy in model which addresses the forward guidance puzzle, and compare it with the commitment policy under rational expectations.

Using the reduced form model, in log-linear representation, these models can be represented in the following way:

$$\begin{aligned} x_t &= -\sigma' \{i_t - \omega E_t \pi_{t+1} - r_t^n\} + M_c E_t x_{t+1} \\ \pi_t &= \kappa' x_t + \beta M_f E_t \pi_{t+1} \end{aligned} \tag{1}$$

Here,  $x_t$  is the output gap,  $\sigma$  is the intertemporal elasticity of substitution,  $i_t$  is the nominal interest rate,  $\pi_t$  is the inflation rate and  $r_t^n$  is the natural rate. Additionally,  $\beta$  is the discount factor and firms update their prices following Calvo pricing, where  $\theta \in (0, 1]$  of the firms keep their prices unchanged. Under rational expectations, we have standard representation with  $\omega = 1$ ,  $\sigma' = \sigma$ ,  $M_c = M_f = \zeta = 1$  and  $\kappa' = \kappa$ , where  $\kappa = \left(\epsilon + \frac{1}{\sigma}\right) \frac{(1-\theta)(1-\beta\theta)}{\theta}$ , and  $\epsilon$  is the Frisch elasticity of substitution. The first equation is referred to as the IS curve and the second as the New-Keynesian Phillips curve (NKPC) in the exposition below. Under Rational Expectations:  $\omega = 1$ ,  $M_c = M_f = \zeta = 1$  and  $\kappa' = \kappa$ .

Under the imperfect common knowledge assumption of Angeletos and Lian (2018), there is a lack of common knowledge about news, and about the beliefs and responses of others. This is interpreted as a form of bounded rationality which is consistent with the rational expectations equilibrium. In terms of the parameters,  $\omega$  is the friction in common knowledge, and in the exposition here, this friction is the same for consumers and firms. With this assumption,  $M_c = \beta + (1 - \beta)\omega \in (\beta, 1]$ ,  $M_f = \theta + (1 - \theta)\omega \in (\theta, 1]$ ,  $\kappa' = \kappa\omega$  and  $\sigma' = \sigma$ , and  $\zeta = 1$ .

With Gabaix's (2020) characterization, the optimizing agents are myopic, and do not anticipate future events perfectly (unlike agents with rational expectations). In this formu-

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<sup>13</sup>The nested model, however, is not exhaustive. Other analyses have proposed mechanisms for reconsidering the effects of forward guidance. Kiley (2016) uses sticky information in price-setting to attenuate the effects of forward guidance. Hagedorn, Luo, Manovskii, and Mitman (2019) examine an NK model with incomplete financial markets in which the central bank's forward guidance may be largely offset by shifts in the distribution of taxes, transfers, and corporate dividends. Farhi and Werning (2019) incorporate k-level thinking and incomplete markets. Lepetit and Fuentes-Albero (2020) incorporate an epidemiology model in a NK model with uninsurable income risk.

lation, the parameter  $\bar{m} \in [0, 1]$  measures "cognitive discounting", that is the attention to the future<sup>14</sup>. With this assumption, the parameters of the reduced form model in (1) are the following:  $M_c = \frac{\bar{m}}{R-r} < 1$ ;  $M_f = \bar{m} \left( \theta + \frac{1-\beta\theta}{1-\beta\theta\bar{m}} (1-\theta) \right) < 1$  and  $\sigma' = \sigma$ .<sup>15</sup> Additionally,  $\omega = \zeta = 1$ .

McKay, Nakamura and Steinsson (2016) obtain a discounting in the aggregate-level Euler equation by introducing a combination of heterogeneity and market incompleteness. This implies that some agents hit their borrowing constraints, thereby violating the individual-level Euler condition. In this case,  $M_c, \zeta < 1$ ;  $M_f = 1$ ;  $\kappa' < \kappa$ , and  $\sigma' = \sigma\zeta$ , where  $\zeta < 1$ . In comparing the formulation in (1) with the representations (2) and (3) in McKay, Nakamura and Steinsson (2017),  $M_c = \alpha$ ,  $\sigma' = \sigma\zeta$ , and  $\kappa'$  corresponds to the slope of the Phillips curve shown in equation A2 of MNS (2017).

The gist of all these models is that discounting is introduced in the aggregate level Euler equation and the NKPC. The parameters for the NK model are assigned values usually used in the literature:  $\sigma$  is 4 (Amato and Laubach, 2003), the slope of the Phillips curve  $\kappa$  is 0.024, and  $\beta = 0.9975$ . The behavioral parameters are set at the values used in the respective papers. In the Angeletos and Lian (2018) specification,  $\omega = 0.75$  and  $M_f = 0.9$ ; in Gabaix (2020),  $M_c = M_f = 0.8$  and in the McKay, Nakamura and Steinsson (2016) specification,  $M_c = 0.97$  and  $\zeta = 0.75$ . The parameter values are summarized in table 2. We now consider these frameworks for characterizing the central bank's policy.

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<sup>14</sup>Using the cognitive discounting parameter, the expectations operation under bounded rationality  $E^{BR}(X_{t+1}) = \bar{m}E^{RE}(X_{t+1})$ , where  $E^{RE}$  is the rational expectations operator for the one-period ahead variable  $X_t$ .

<sup>15</sup>We are considering the version for Gabaix's model in which the other cognitive discounting factors, such as  $\bar{m}_y$  for income and  $\bar{m}_r$  for the interest rate are set at their rational expectations values of 1. This implies that the individual level Euler equations are not "distorted".

**Table 2: Benchmark model parameters**

<i>Name</i>	<i>Param</i>	<i>Value</i>
IES	$\sigma$	4
Phillips curve slope	$\kappa$	0.024
Discount factor	$\beta$	0.9975
Natural rate, AR(1)	$\rho$	0.94
Natural rate	$r_n^{shock}$	-0.01
Angeletos and Lian	$\omega$	0.75
parameters	$M_f$	0.9
Gabaix	$M_c$	0.8
parameters	$M_f$	0.8
McKay, Nakamura	$M_c$	0.97
and Steinsson	$\zeta$	0.75
parameters		

## 4 Optimal Policy with Full Credibility

We present here the model with full credibility and the corresponding quantitative analysis.

### 4.1 Model Framework

In the following, we present the results for characterizing optimal policy under commitment at the zero-lower bound.<sup>16</sup> The natural rate shock is assumed to follow an AR(1) process. Following the literature, the central bank’s problem is to choose the path of  $\{i_0, i_1, \dots\}$  to minimize the loss function  $L_t = \pi_t^2 + \lambda x_t^2$ :

$$E_0 \sum_{t=0}^{\infty} \beta^t L_t \tag{2}$$

subject to the IS and aggregate supply equations in (1). Here  $\beta$  is the discount factor, and  $\lambda$  is the positive parameter denoting the weight assigned by the central bank to output stability.

In the case where the central bank commits to a course of action, it’s problem is to choose a state contingent sequence for  $x_t$  and  $\pi_t$  to maximize the objective in (2) assuming that equations in (1) hold in every period  $t$ . As noted by Clarida, Gali and Gertler (1999;

<sup>16</sup>We show the solution to the central bank’s problem under discretion in appendix B.

CGG henceforth), the central bank recognizes that the private sector will be forming its expectations based on the evolution of  $x_t$  and  $\pi_t$ , and therefore does not treat them as fixed while solving the optimization problem.

Under the commitment strategy, the first order conditions for the central bank's problem are:

$$\frac{\partial \mathcal{L}}{\partial x_t} : -\phi_{1t-1} \frac{M_c}{\beta} + (\lambda x_t + \phi_{1t} - \phi_{2t} \kappa') = 0 \quad (3a)$$

$$\frac{\partial \mathcal{L}}{\partial \pi_t} : \left( -\phi_{1t-1} \frac{\sigma \omega}{\beta} - \phi_{2t-1} M_f \right) + (\pi_t + \phi_{2t}) = 0 \quad (3b)$$

$$i_t \phi_{1t} = 0; i_t \geq 0, \phi_{1t} \geq 0 \quad (3c)$$

Assuming initial conditions of  $\phi_{1-1} = \phi_{2-1} = 0$ , the solution to the system is obtained from the above first order conditions, and the the IS and NKPC curves. Here we assume that  $i_t$  is bounded at the ELB until  $T_c^*$ .

After  $T_c^*$ , the Lagrange multiplier  $\phi_{1t}$  for  $t > T_c^*$  is zero. For  $t = T_c^* + 2, \dots$ , there is a unique bounded solution, with the dynamic path given by:

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \begin{bmatrix} (1 - \mu_1) \\ \frac{\kappa'}{\lambda} \mu_1 \end{bmatrix} \phi_{2t-1}, \quad (4)$$

$$\phi_{2t} = \mu_1 \phi_{2t-1} \quad (5)$$

where  $\mu_1$  is the eigenvalue of the matrix solved for in the appendix. Finally, the path of  $i_t$  is given by:

$$\sigma i_t = -x_t + M_c E_t x_{t+1} + \omega E_t \pi_{t+1} + r_t^n \quad (6)$$

$$= \mu_1 \left[ -\frac{\kappa'}{\lambda} (1 - M_c \mu_1) + \omega (1 - \mu_1) \right] \phi_{2t-1} + r_t^n \quad (7)$$

Thus, under commitment, the path of  $i_t$  now depends on the extent to which knowledge is "imperfect".

For the period during the zero lower bound, we set  $i_t = 0$  in the IS, aggregate supply and first order conditions. Then,

$$A = \begin{bmatrix} \beta M_f + \kappa' \sigma \omega & \kappa' M_c \\ \sigma \omega & M_c \end{bmatrix}, \text{ and } a = \begin{bmatrix} \kappa' \sigma \\ \sigma \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = A \begin{bmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} + a r_t^n \quad (9)$$

Using the terminal condition at  $T_c^* + 1$  we get

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \sum_{j=t}^{T_c^*} A^{-(j-t-1)} a r_j^n + A^{-(T_c^*-t+1)} \begin{bmatrix} \pi_{T_c^*+1} \\ x_{T_c^*+1} \end{bmatrix} \quad (10)$$

and,

$$\begin{bmatrix} \phi_{1t} \\ \phi_{2t} \end{bmatrix} = \begin{bmatrix} \frac{M_c}{\beta} + \kappa' \frac{\sigma\omega}{\beta} & \kappa' M_f \\ \frac{\sigma\omega}{\beta} & M_f \end{bmatrix} \begin{bmatrix} \phi_{1t-1} \\ \phi_{2t-1} \end{bmatrix} + \begin{bmatrix} -\kappa' & -\lambda \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \end{bmatrix} \quad (11)$$

Then, for  $T_c^* + 1$ ,  $\phi_{1T_c^*+1} = 0$ . Substituting into the FOCs under commitment, along with the NKPC:

$$\pi_{T_c^*+1} + \phi_{2T_c^*+1} = \phi_{1T_c^*} \frac{\sigma\omega}{\beta} + \phi_{2T_c^*} M_f \quad (12)$$

$$\lambda x_{T_c^*+1} - \phi_{2T_c^*+1} \kappa' = \phi_{1T_c^*} \frac{M_c}{\beta} \quad (13)$$

$$\pi_{T_c^*+1} - \kappa' x_{T_c^*+1} = \beta M_f E_{T_c^*+1} \pi_{T_c^*+2} \quad (14)$$

Then, the equations (4), (6), (10), (11) and (12), along with  $\phi_{1-1} = \phi_{2-1} = 0$ , and  $i_t = 0$  for  $t = 0, \dots, T_c^*$  and  $\phi_{1t} = 0$  for  $t = T_c^* + 1, \dots$  characterize the solution under commitment.

## 4.2 Quantitative Analysis

Figure 4 compares these alternative specifications under the benchmark assumption that the central bank's policy strategy is completely transparent and credible. In particular, we consider a stylized experiment in which the natural rate of interest drops sharply in the initial period and then gradually reverts towards its steady-state value. In particular, the natural rate follows an AR(1) process,  $r_t^n = \rho_r r_{t-1}^n + \varepsilon_t$ , with  $\rho_r = 0.94$  and a shock size of  $-0.01$ . With these parameters, the natural rate remains negative for 23 quarters before turning positive<sup>17</sup>. In effect, this shock represents a scenario involving a large and persistent shortfall in aggregate demand, perhaps roughly similar in magnitude to the impact of the global financial crisis on the U.S. economy.

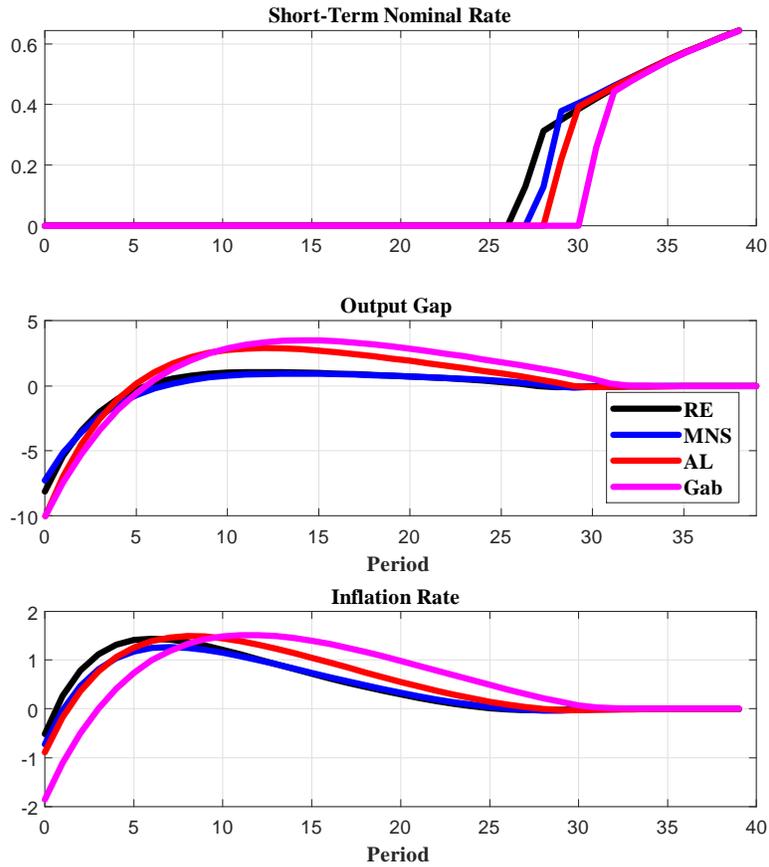
<sup>17</sup>According to the November 2020 minutes of the FOMC, the respondents from the Survey of Primary Dealers and Survey of Market Participants indicate that the committee will "most likely start raising the target range for the federal funds rate...around 2024." We use this approximate length of time to calibrate the size of the shock. We have also performed sensitivity analysis using other shock sizes, and the results are available upon request.

The optimal monetary policy involves a commitment to a “lower-for-longer” strategy, so that the actual nominal interest rate is pinned at the ELB for an additional year or two after the natural rate becomes positive. Even with this policy in place, the initial impact of the shock is severe: The output gap is about  $-8\%$ , and inflation falls noticeably below target. Nonetheless, the optimal monetary policy induces a rapid recovery involving a persistent boom in output and an elevated level of inflation over the subsequent half-decade. For each of the model specifications, the optimal policy is markedly more aggressive than a pure make-up strategy.<sup>18</sup> In particular, the initial shortfall of inflation is relatively modest and transitory, while the overshooting of inflation is substantial (about one percentage point above target) and persistent (only subsiding after about five years).

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<sup>18</sup>Bernanke, Kiley and Roberts (2019) and Reifschneider and Wilcox (2019) performed simulations of FRB/US and reached similar conclusions about the shortcomings of pure make-up strategies in mitigating the ELB.

Figure 4: Optimal Policy with Full credibility



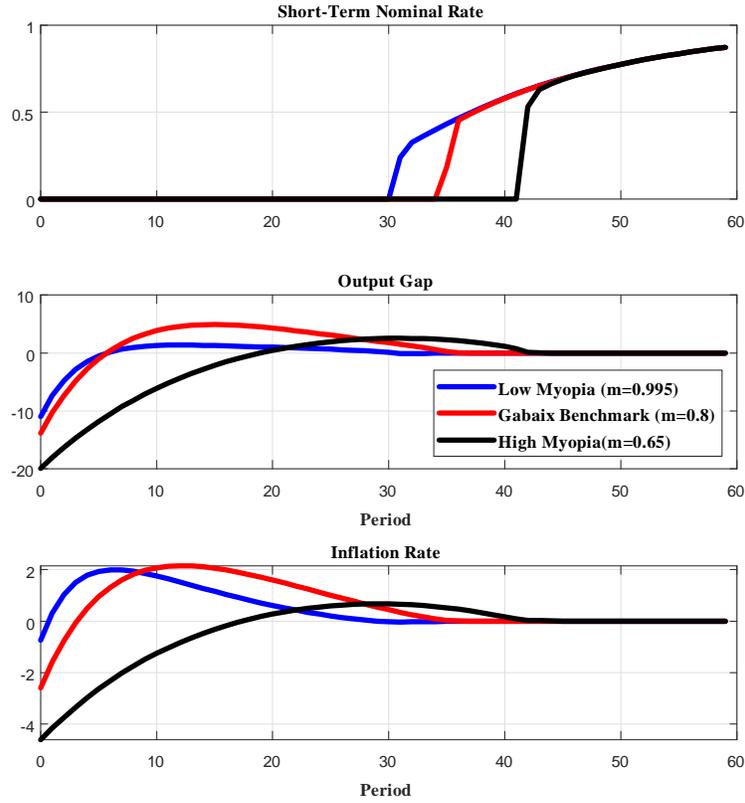
Note: These figures show the responses of the four models (rational expectations, RE; McKay, Nakamura and Steinsson, MNS; Angeletos and Lian, AL and Gabaix, Gab) in response to a natural rate shock. The figure shows the evolution of the short nominal interest rate, output gap and inflation across the various model specifications. Time periods are denoted in quarters. The inflation rate is expressed in annual rates in percent; the interest rate and output gap are expressed in percentage points.

A commitment to a more aggressive “lower-for-longer” strategy could be reasonably effective in mitigating the ELB if such a commitment were fully transparent and credible. Before reaching any definitive conclusions, however, it would be sensible to examine the performance of such strategies in larger-scale DSGE models that incorporate a range of alternative specifications regarding the private sector’s expectations formation.

In figure 5 we consider the extent to which myopia matters for the optimal policy effects. The benchmark level of the myopia parameter corresponds to the slope of the Phillips curve in (1) to be approximately 0.8, which corresponds to Gáli and Gertler (1999). A higher degree of myopia, with  $M_f = 0.65$ , would correspond to the approximate value of the Phillips curve slope found in Fuhrer and Rudebusch (2004). We also consider a larger shock, with  $r_{shock}^n = -0.0125$  while keeping the same shock persistence. At this higher degree of myopia, the length of the ELB is extended by more than a full year compared to the benchmark case.

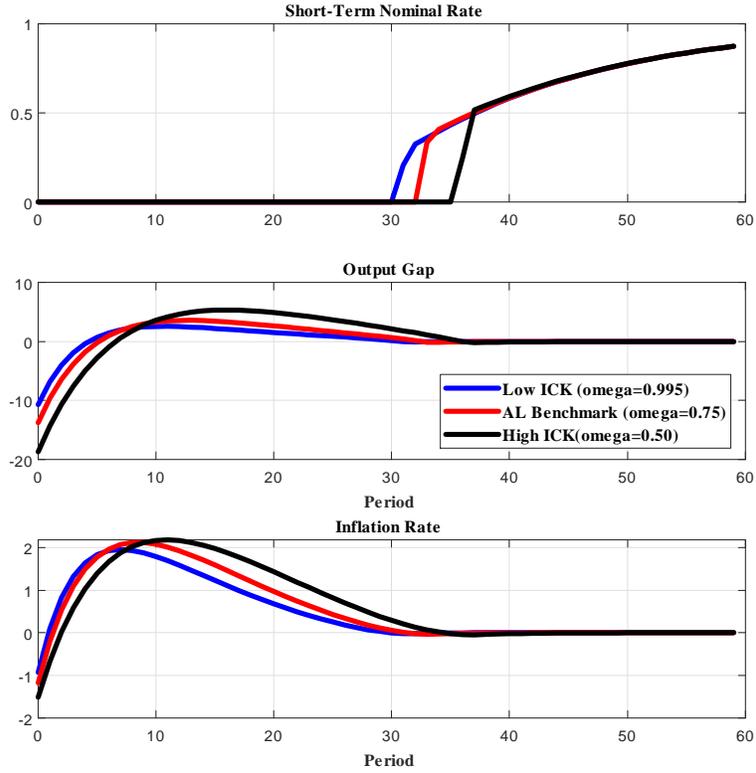
Figure 6 shows the responses of the economy with different levels of imperfect common knowledge. The benchmark parameter of  $\omega = 0.75$  corresponds to the case where every agent who has heard the policy announcement believes that another agent failed to believe the policy with a probability of 25%. As Angeletos and Lian (2018) discuss, a greater degree of imperfect common knowledge with  $\omega = 0.5$  would align with the evidence of Coibion and Gorodnichenko (2012). This leads to an even larger shortfall of inflation from its target.

Figure 5: Optimal Policy with Varying Myopia



Note: These figures show the responses of the Gabaix model in response to a natural rate shock with varying levels of myopia. Here, the persistence of the shock  $\rho = 0.94$  and  $r_{shock}^n = -0.0125$ . The figure shows the evolution of the short nominal interest rate, output gap and inflation across the various model specifications. Time periods are denoted in quarters. The inflation rate is expressed in annual rates in percent; the interest rate and output gap are expressed in percentage points.

Figure 6: Optimal Policy with Varying Imperfect Common Knowledge



Note: These figures show the responses of the Angeletos and Lian model in response to a natural rate shock with varying levels of imperfect common knowledge. Here, the persistence of the shock  $\rho = 0.94$  and  $r_{shock}^n = -0.0125$ . The figure shows the evolution of the short nominal interest rate, output gap and inflation across the various model specifications. Time periods are denoted in quarters. The inflation rate is expressed in annual rates in percent; the interest rate and output gap are expressed in percentage points.

In table 3, we illustrate the stabilization properties of the different models for alternative values of key NK parameters:  $\sigma$ ,  $\kappa$  and the natural rate shock. The first row of the table shows results using the benchmark parameters. The alternative value of  $\sigma$  considered is taken from Eggertsson and Woodford (2003). The stabilization properties of the model improve, but the time that the central bank has to commit to staying at the ELB increases significantly. In case of a more persistent shock (the last row), the economy is forced to stay at the ELB for more than twelve years.

**Table 3: Varying model parameters**

<i>Param</i>	<i>T</i>	$\sigma(\pi)$	$\sigma(x)$	<i>T</i>	$\sigma(\pi)$	$\sigma(x)$	<i>T</i>	$\sigma(\pi)$	$\sigma(x)$	<i>T</i>	$\sigma(\pi)$	$\sigma(x)$
	Rational Exp			Gabaix			AL			MNS		
BM	28	0.6	7.1	32	0.8	12.9	30	0.6	10.3	29	0.5	6.6
$\sigma = 0.5$	37	0.2	2.7	34	0.2	2.1	38	0.2	3.1	38	0.2	2.1
$\rho = 0.98,$ $r_n^{shock} = -0.006$	49	0.3	3.2	53	0.5	7.2	50	0.4	5.2	50	0.3	3.0

Note: The first column denotes model parameters being varied. The benchmark model parameters are in the row labeled "BM". The number of periods for which the ELB is maintained under the commitment strategy is denoted by  $T$ . The standard deviations of inflation and the output gap are shown in the  $\sigma(\pi)$  and  $\sigma(x)$  columns. The panel for rational expectations is first, followed by the results of the Gabaix (2020), Angeletos and Lian (2018) and finally the McKay, Nakamura and Steinsson (2016) models.

## 5 Imperfect Credibility

Now we turn to scenarios in which the central bank has more limited credibility, especially with regard to policy commitments that extend over a multi-year timeframe. The challenge of imperfect credibility has been readily apparent from the historical record on disinflationary episodes. Indeed, as emphasized by the landmark study of Bernanke, Laubach, Mishkin, and Posen (2001), the mere announcement of an inflation target had little or no effect on actual inflation in several advanced economies. In such cases, however, the central bank can start gaining credibility immediately by tightening the stance of monetary policy at the start of the disinflation and then easing gradually as actual and expected inflation move downwards towards the target.

By contrast, gaining credibility may be particularly difficult when the economy undergoes a persistent shortfall in aggregate demand that pins the nominal interest rate at the ELB over

a protracted period. In such circumstances, the central bank may emphasize its intention to follow a “lower-for-longer” strategy once the ELB is no longer binding, but policymakers have no practical means of earning credibility up-front by taking immediate action to demonstrate their commitment to the strategy. Indeed, the commitment remains completely vacuous as long as the natural rate of interest remains below the ELB and cannot be put into practice until the natural rate rises above that threshold.

As noted in the FOMC minutes<sup>19</sup>, the effectiveness of a make-up strategy would depend on "the private sector's understanding of the strategy and on their confidence that future policymakers would follow through on promises to keep policy accommodative." The FOMC meeting transcripts from 2011-12 indicate that policymakers at that time were acutely aware that longer-horizon forward guidance could be impaired by intrinsic limits on the FOMC's ability to make credible promises on behalf of future members.

These concerns are amplified in the downturn of 2020. As noted in the minutes of the April 29, 2020 meeting, "Yields on nominal Treasury securities declined across the maturity spectrum, with the 10- and 30-year yields ending the period near all-time lows. A straight read of market quotes suggested that the expected federal funds rate would remain under 25 basis points through 2022." In figure 1, we show the term structure of forward rates for the 2-, 5- and 10-year nominal yields for the July-December 2007 and January-April 2020 periods to illustrate the point that financial market expectations are of a prolonged period of low interest rates.

## 5.1 Analytical Framework

In light of these considerations, we now analyze the optimal monetary policy when its credibility depends on the length of time over which policy is pinned at the ELB. In particular, we assume that the private sector perceives a risk that in any given period the central bank may renege on its prior commitment and revert to a purely discretionary policy, that is, the central bank would simply adjust the nominal interest rate in line with the natural rate of interest once the ELB was no longer binding. The likelihood of revocation is represented by a Poisson process with a constant hazard rate. For simplicity, we assume that the central bank's commitment is fully credible once the ELB is no longer a binding constraint, because from that point onwards the private sector can directly observe that the “lower-for-longer” strategy is being implemented. It should also be noted that the central bank is fully cognizant of its own imperfect credibility and takes that into account in formulating its optimal policy strategy.

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<sup>19</sup>FOMC Minutes, September 2019.

The central bank is assumed to understand that the agents perceive it to have imperfect credibility. From periods  $t = 0$  to  $T'$ , the agents assign a positive probability  $(1 - \Theta)$  to the central bank reneging its commitment, and following the discretion solution. In this case, the loss function is minimized with respect to:

$$x_t = M_c (\Theta E_t x_{t+1} + (1 - \Theta) E_t x_{t+1}^D) \quad (15a)$$

$$-\sigma' \{i_t - \omega (\Theta E_t \pi_{t+1} + (1 - \Theta) E_t \pi_{t+1}^D) - r_t^n\} \quad (15b)$$

$$\pi_t = \kappa' x_t + \beta M_f (\Theta E_t \pi_{t+1} + (1 - \Theta) E_t \pi_{t+1}^D) \quad (15c)$$

The additional terms in the IS and aggregate supply equations corresponding to  $\Theta$  and  $(1 - \Theta)$  indicate that the optimizing households only assign a probability of  $\Theta$  that the central bank will actually follow through on its commitment strategy. In the perfect credibility case above,  $\Theta = 1$ .

The natural rate shock dissipates in period  $T'$ . In the following period, from  $T' + 1$  onwards, once the agents observe that the central bank has maintained the commitment path, even after the natural rate shock has dissipated, the imperfect credibility is resolved. Then, the relevant IS and aggregate supply equations are again given by (1). Lift-off from the zero-lower bound is assumed to occur in some period  $T^* > T' + 1$ . In order to illustrate the model implications arising from this formulation of imperfect credibility, we show the first order conditions with respect to the output gap here, and the conditions with respect to inflation are noted in the appendix:

FOCs with respect to  $x_t$  at  $t = T' + 1$  :

$$\frac{\partial \mathcal{L}}{\partial x_t} = \beta^{T'} \{2\phi_{1T'} (-M_c \Theta)\} + \beta^{T'+1} \{2\lambda x_{T'+1} + 2\phi_{1T'+1} - 2\phi_{2T'+1} \kappa'\} = 0 \quad (16)$$

FOCs with respect to  $x_t$  at  $t = T' + 2$  :

$$\frac{\partial \mathcal{L}}{\partial x_t} = \beta^{T'} \{2\phi_{1T'+1} (-M_c)\} + \beta^{T'+1} \{2\lambda x_{T'+2} + 2\phi_{1T'+2} - 2\phi_{2T'+2} \kappa'\} = 0 \quad (17)$$

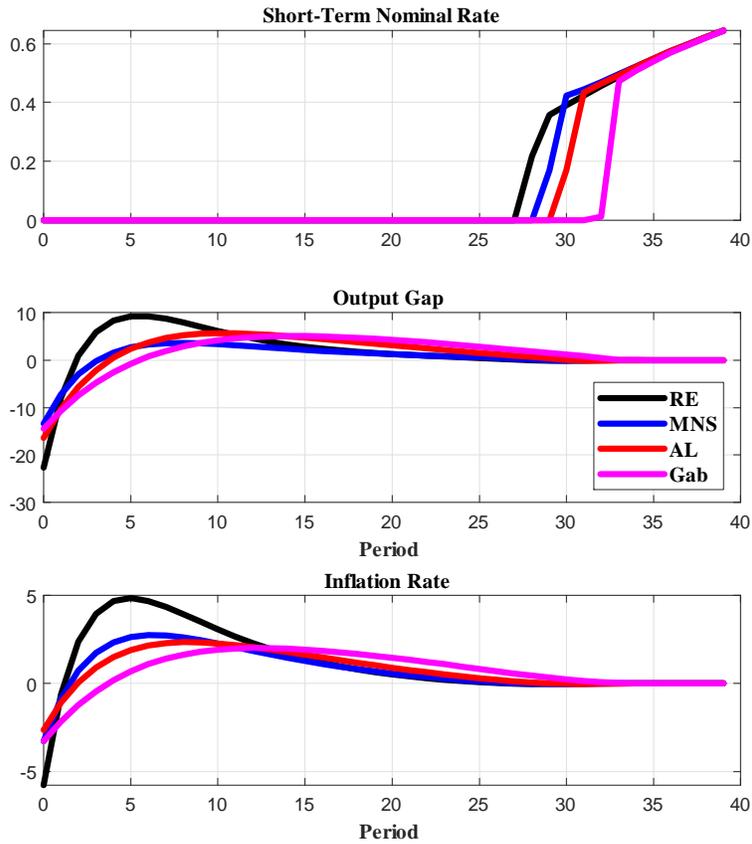
As shown in the discretion solution (21) in appendix C, since the output gap and inflation variables are functions of the exogenous natural rate, the terms corresponding to the discretion solution do not appear in the FOCs.

## 5.2 Quantitative Analysis

We calibrate the degree of imperfect credibility so that the probability of renegeing is perceived to be 2.5 percent in any given quarter. Thus, in our benchmark scenario where the ELB is binding for about seven years, the private sector initially perceives 50/50 odds that the central bank will follow through with its commitment to the “lower-for-longer” strategy. Those perceived odds rise gradually over time as the central bank continues to reiterate its commitment and the time approaches when the commitment will be implemented. This calibration seems broadly consistent with the concerns flagged in past FOMC discussions about the extent to which its composition evolves gradually over time as a result of staggered terms, retirements, and other sources of turnover.

As shown in figure 7, the timing of liftoff from the ELB under imperfect credibility is roughly similar to that implied by perfect credibility (as shown previously in figure 4). Interestingly, the deterioration in macroeconomic stabilization is most severe in the baseline model with rational expectations. As emphasized in various studies of the forward guidance puzzle, this model exhibits a very strong feedback loop: when the nominal interest rate is pinned to the ELB, a decline in inflation raises the ex ante real interest rate, which in turn exerts further downward pressure on real output and inflation. Consequently, when faced with imperfect credibility, the central bank adopts a more aggressive “lower-for-longer” strategy, inducing a huge economic boom that helps dampen the initial downturn. In particular, the output gap initially plummets to around  $-20\%$  and then rebounds to around  $+10\%$ , while inflation initially drops  $5\%$  below target and then surges  $5\%$  above target.

Figure 7: Optimal Policy with Imperfect Credibility



Note: These figures show the responses of the four models (rational expectations, RE; McKay, Nakamura and Steinsson, MNS; Angeletos and Lian, AL and Gabaix, Gab) in response to a natural rate shock. The figure shows the evolution of the short nominal interest rate, output gap and inflation across the various model specifications. Time periods are denoted in quarters. The inflation rate is expressed in annual rates in percent; the interest rate and output gap are expressed in percentage points.

Nonetheless, imperfect credibility induces a marked deterioration in macroeconomic stability, regardless of the particular specification of expectations formation. For example, in the specification of Angeletos and Lian (2018), the output gap drops sharply to around  $-15\%$  and then exhibits a sustained boom of around  $5\%$ , while inflation initially falls  $2\%$  below target and then overshoots the target by  $2\%$  for several years.

In table 4, we show the stabilization properties of the different model for varying degrees of credibility. Relative to the benchmark case of  $\Theta = 0.975$ , a lower value of  $0.95$  implies that the probability of renegeing rises to  $5\%$  in every quarter, so that after five years, the economy is only placing a  $35\%$  probability on the central bank following through with its commitment. As can be seen from the last row of table 3, the stabilization properties are markedly worse in terms of output stabilization.

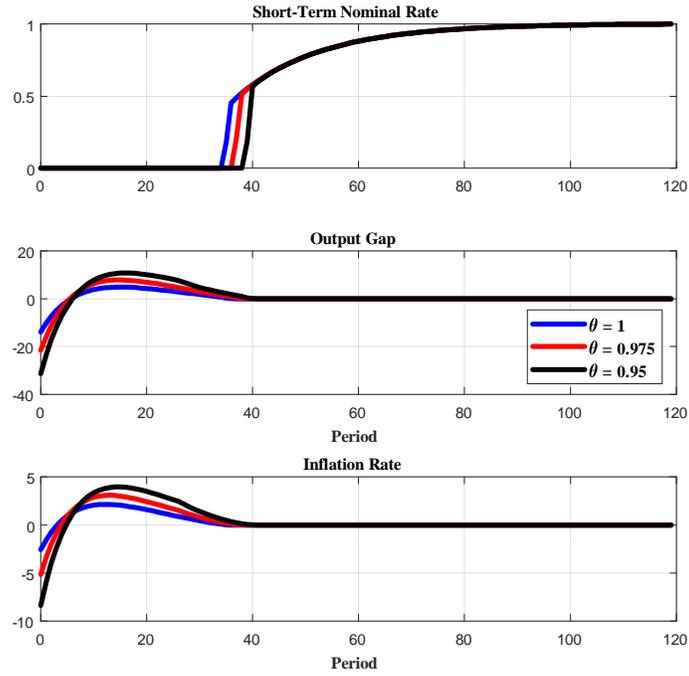
**Table 4: Varying the credibility parameter**

$\Theta$	$T$	$\sigma(\pi)$	$\sigma(x)$	$T$	$\sigma(\pi)$	$\sigma(x)$	$T$	$\sigma(\pi)$	$\sigma(x)$	$T$	$\sigma(\pi)$	$\sigma(x)$
	Rational Exp			Gabaix			AL			MNS		
1	28	0.6	7.8	32	0.8	12.2	30	0.6	10.8	29	0.5	7.0
0.975	29	2.1	22.6	33	1.2	17.6	31	1.1	17.5	30	1.2	12.5
0.950	30	3.9	42.4	35	1.7	24.3	32	1.7	26.2	31	2.1	20.4

Note: The first column denotes the credibility parameter  $\Theta$ . Perfect credibility corresponds to  $\Theta = 1$ . The number of periods for which the ELB is maintained under the commitment strategy is denoted by  $T$ . The standard deviations of inflation and the output gap are shown in the  $\sigma(\pi)$  and  $\sigma(x)$  columns. The panel for rational expectations is first, followed by the results of the Gabaix (2020), Angeletos and Lian (2018) and finally the McKay, Nakamura and Steinsson (2016) models.

To illustrate the significance of imperfect credibility in the case of a large macroeconomic shock, we consider a shock with  $r_{shock}^n = -0.0125$  and the same persistence as the benchmark case, in figure 8. Here we show the responses using the Gabaix (2020) model. As illustrated, in the case of the benchmark level of imperfect credibility, the initial deflation of  $5\%$  is followed by inflation of more than double its target of  $2\%$  for approximately 20 quarters.

Figure 8: Optimal Policy under Gabaix Model with Great Depression Shock



Note: These figures show the responses of the Gabaix model in response to a natural rate shock which is a large, Great Depression-style shock. Here, the persistence of the shock  $\rho = 0.94$  and  $r_{shock}^n = -0.0125$ . The figure shows the evolution of the short nominal interest rate, output gap and inflation. Time periods are denoted in quarters. The inflation rate is expressed in annual rates in percent; the interest rate and output gap are expressed in percentage points.

## 6 Model Uncertainty

As in most previous studies of monetary policy at the ELB, the foregoing analysis in this paper has assumed that policymakers have a complete understanding of the true dynamic structure of the economy, as captured by a specific macroeconomic model. Under this admittedly heroic assumption, it is reasonably straightforward to determine the monetary policy strategy that provides optimal stabilization outcomes in that particular model.

Nonetheless, it has long been recognized that an appropriate monetary policy strategy should provide robust performance in the face of model uncertainty; see McCallum (1988), Taylor (1993, 1999), and Hansen and Sargent (2001). That literature has underscored the pitfalls of policy strategies that hinge on the accuracy of longer-horizon forecasts that are finetuned to the characteristics of a specific model.<sup>20</sup>

To illustrate these considerations in the present context, we now focus on a particular aspect of the NK model that is a subject of ongoing analysis and debate, namely, the specification of the New Keynesian Phillips Curve (NKPC). For the analysis shown in the previous figures, we calibrated the NKPC slope coefficient using the estimate obtained by Amato and Laubach (2003). Now we consider the possibility of an even flatter NKPC, with a slope coefficient of 0.01 – a bit less than half the slope of 0.024 in our baseline calibration. For simplicity, we conduct this analysis using the model of Angeletos and Lian (2018), and we assume that the central bank’s policy strategy is fully transparent and credible to the private sector. The results are shown in figure 9.

If the central bank knows that the NKPC is very flat (as denoted by the solid blue line in each panel), then the optimal policy prescribes a protracted period of about nine years at the ELB, that is, about two years longer than in the baseline calibration shown in figure 4. That policy reflects the fact that a flatter NKPC attenuates the feedback loop noted above, namely, a shift in the output gap has muted effects on inflation and hence induces a smaller movement in the real interest rate. Consequently, this optimal policy is associated with a deeper recession of around  $-15\%$ , a more modest initial decline in inflation, and a shallow but highly persistent phase of overshooting thereafter.

Now we consider the scenario in which the NKPC actually has a relatively flat slope of 0.01 but the central bank incorrectly formulates its “optimal” policy strategy based on a steeper slope of 0.024. In effect, the central bank’s strategy mistakenly embeds a relatively strong feedback loop between the output gap, the inflation rate, and the real interest rate, and that misperception results in a dramatic set of policy errors: the nominal interest rate remains at the ELB for nearly 50 quarters, inducing a huge and persistent boom in which

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<sup>20</sup>See Levin, Wieland, and Williams (1999, 2003) and Levin and Williams (2003).

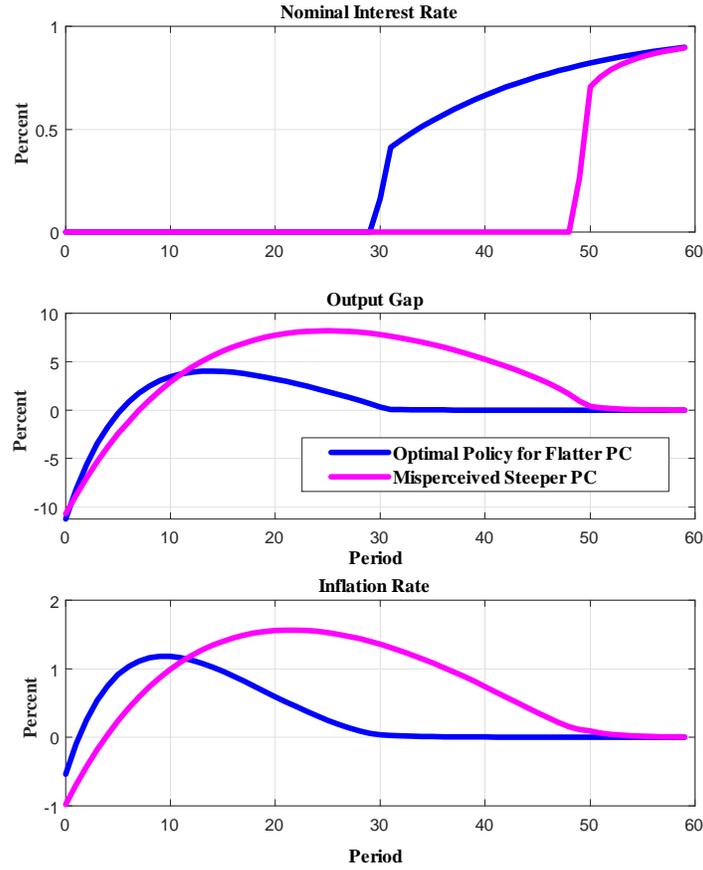
the output gap peaks at nearly 10%. The deviation from price stability is somewhat milder due to the actual flatness of the NKPC; nonetheless, the inflation rate is elevated by more than 1% above target for nearly a decade. This is shown in figure 9.

Of course, this exercise is merely illustrative, involving a single parameter in a small stylized NK model. In practice, policymakers face a high degree of uncertainty not only about the determination of inflation but about many other aspects of the economy. Indeed, the minutes of the September 2019 FOMC meeting indicate that a number of participants referred to the staff’s analysis of the FRB/US model and highlighted “the need for more robustness analysis of simulation results along several dimensions and for further comparison to other alternative strategies.”<sup>21</sup>

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<sup>21</sup>FOMC Minutes, September 18-19, 2019, p.4. Available at: <https://www.federalreserve.gov/monetarypolicy/fomcminutes20190918.htm>.

Figure 9: Optimal Policy and Model Uncertainty



Note: These figures show the responses of the Angeletos and Lian (2018) model, in response to a natural rate shock, when there is uncertainty about the slope of the Phillips curve. The figure shows the evolution of the short nominal interest rate, output gap and inflation. Time periods are denoted in quarters. The inflation rate is expressed in annual rates in percent; the interest rate and output gap are expressed in percentage points.

## 7 Conclusions

In this paper, we have assessed the performance of optimal monetary policy strategies at the ELB under alternative assumptions about expectations formation, imperfect credibility, and model uncertainty. We find that imperfect credibility induces a significant deterioration in macroeconomic stability, regardless of the particular specification of expectations formation. Consequently, the net benefits of forward guidance are much more tenuous than in settings with rational expectations and perfect credibility.

Of course, this analysis has been conducted using a prototypical New Keynesian model. In future research, it would be helpful to investigate these issues using larger and more empirically plausible models.<sup>22</sup> For example, McKay and Wieland (2020) specifically consider the attenuated efficacy of forward guidance in a model with durable consumer goods. It would also be helpful to incorporate mechanisms in which private agents learn over time about the true structure of the economy.<sup>23</sup>

Finally, these results underscore the distinction between Delphic vs. Odyssean forward guidance as identified by Campbell, Evans, Fisher, and Justiniano (2012). In the wake of a severe shock like the COVID-19 pandemic, Delphic forward guidance about the path of interest rates over the next several years merely underscores the protracted nature of the economic recovery. In that context, efforts to issue Odyssean forward guidance may well be ineffectual or even counterproductive, simply reinforcing the private sector’s pessimism about the economic outlook rather than boosting the degree of monetary stimulus. Moreover, if attempts to provide Odyssean forward guidance are perceived as ineffective, such perceptions may exacerbate the central bank’s temptation to renege on its promised overshooting, i.e., its imperfect credibility could become a self-fulfilling prophecy.

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<sup>22</sup>See Smets and Wouters (2007); Blanchard and Gali (2007). In this framework, capital is a factor of intermediate goods production and in addition to price stickiness, the model also features nominal wage stickiness. The model also includes investment adjustment costs, habit formation in consumption, and partial dynamic indexation of prices and wages to lagged values to generate a richer autocorrelation structure.

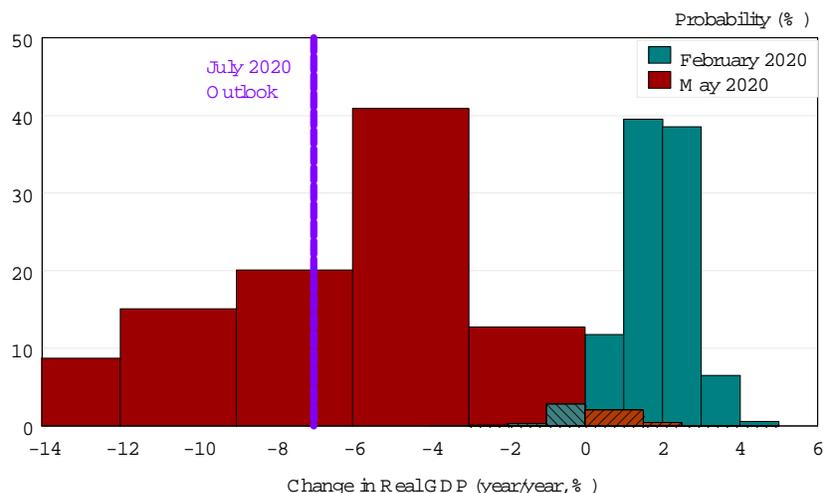
<sup>23</sup>See Fuhrer (1996), Eusepi and Preston (2011), Sinha (2016).

# Appendix

## Appendix A: Results from the Survey of Primary Dealers

Here we consider the myopia of forecasters of the SPD. We consider three surveys in 2020. In the March 2020 survey, the forecasters placed an approximately 30% probability that the Q4-over-Q4 growth in the real GDP would be negative. In the April 2020 survey, the mass of the distribution began to shift to a more pessimistic outlook, but it was only in the June 2020 survey that this view became more entrenched.

Figure A1: Myopia among SPD Forecasters

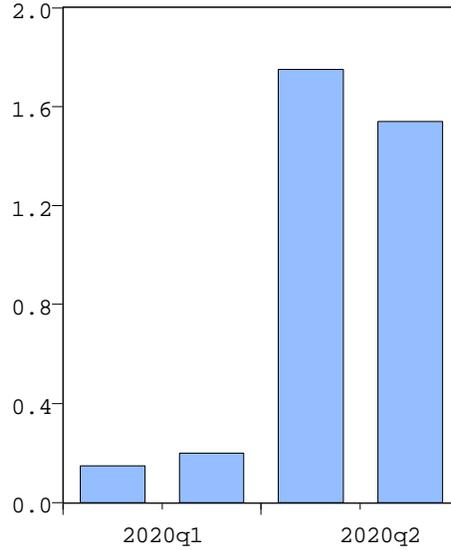


Note: These are the probabilities assigned by the SPD forecasters to the Q4-over-Q4 change in the real GDP. The surveys were conducted in 2020 on the dates indicated.

Figure A2 shows the dispersion in forecasts among the SPD participants for the 2020 surveys. As with the SPF, we find that this dispersion measure was the highest on record for the series. It is also worth noting that there was strikingly little disagreement among the forecasters in SPD in March 2020, even though the Covid-19 crisis had begun to unfold at a rapid pace outside the United States, and the Federal Reserve has already initiated the 50 b.p. reduction in the Federal Funds Rate<sup>24</sup>. The SPF also indicates the same low level of disagreement in the first quarter of 2020, but since that data is collected primarily in the middle of the quarter, it may be less indicative of the changing economic situation.

<sup>24</sup>The survey questions were distributed on March 4, 2020 and the responses were received on March 9, 2020. The FFR reduction took place on March 3, 2020.

Figure A2: Dispersion among SPD Forecasters



Note: These are the Interquartile Ranges (IQRs) reported by the forecasters in the Federal Reserve Bank of New York’s Survey of Primary Dealers. The Real GDP responses are the 1-year ahead Q4/Q4 growth estimates. The dates indicate the time when the surveys were received by the FRBNY. The non-recession mean is noted by the green line.

In the following tables, we present the tables referred to in section 2.2.1 of the paper.

**Table A1: Question Format from the Federal Reserve Bank of New York’s Primary Dealer Survey**

*Of the possible outcomes below, please indicate the percent chance you attach to the timing of the first federal funds target rate increase.*

	Q2 2011	Q3 2011	Q4 2011	Q1 2012	Q2 2012	Q3 2012	Q4 2012	Q1 2013	≥Q2 2013
<i>Average</i>	1%	3%	12%	20%	19%	17%	13%	8%	7%
Most likely quarter and year of first target rate increase:									
25th percentile	Q1 2012								
Median	Q2 2012								
75th percentile	Q4 2012								

Note: These are the survey responses from the January 2011 survey. The survey was distributed on January 13, 2011 and it was received by January 18, 2011.

**Table A2: Change in Median Lift-off Dates from the FRBNY Survey**

<i>Date of Survey</i>	<i>Median Liftoff Date</i>	$\Delta$ <i>Median Liftoff Date</i>
Jan 13, 2011	2012Q3	-
Mar 7, 2011	2012Q3	0
Apr 18, 2011	2012Q2	-1
Jun 13, 2011	2012Q3	1
Aug 1, 2011	2012Q4	1
Sep 12, 2011	2013Q4	0
Oct 24, 2011	2014Q1	1
Dec 5, 2011	2014Q2	1
Jan 17, 2012	2014Q2	0
Apr 16, 2012	2014Q3	1
Jun 11, 2012	2014Q4	1
Jul 23, 2012	2015Q1	1
Sep 4, 2012	2015Q3	2
Oct 15, 2012	2015Q3	0
Dec 3, 2012	2015Q3	0
Dec 17, 2012	2015Q3	0

Note: These are the change in the median expectations of primary dealers about the lift-off date. The dates indicate the day on which the survey was received by FRBNY.

**Table A3: Change in Nominal and Real Yields around FOMC Forward Guidance Dates**

<i>FG Dates</i>	$\Delta$ <i>Nom1yr</i>	$\Delta$ <i>Nom2yr</i>	$\Delta$ <i>Nom5yr</i>	$\Delta$ <i>TIPS2yr</i>	$\Delta$ <i>TIPS5yr</i>
Dec 16, 2008	-8.0	-10.7	-16.3	-35.8	-28.9
Mar 18, 2009	-17.3	-26.4	-47.1	-52.1	-58.2
Aug 9, 2011	-3.7	-8.6	-19.1	-15.1	-21.2
Jan 1, 2012	-0.2	-3.8	-9.4	-11.6	-13.6
Sep 13, 2012	-0.2	-0.9	-3.7	-16.0	-15.5
Dec 12, 2012	-0.3	0.0	2.2	-2.3	0.3

Note: These are the changes, in basis points, for the 1, 2 and 5 year nominal yields and the 2 and 5 year TIPS yields. The yields are obtained from Gürkaynak, Sack and Wright (2007 and 2010), available on the Federal Reserve Board website.

## Appendix B: Central Bank's Problem under Discretion

Here we present the central bank's problem under the discretion case.

In order to quantify the gains from its commitment strategy, we present the central bank's problem under the discretion case here. In this setting, the central bank chooses  $(x_t, \pi_t, i_t)$  each period to minimize the function in (2) subject to the equations in (1). The central bank takes private sector expectations as given in the absence of commitment since it is only

choosing the values of  $x_t$  and  $\pi_t$  for the current period. Then the central bank's problem is reduced a series of static optimizations to minimize:

$$[\pi_t^2 + \lambda x_t^2] + F_{t+1}$$

where  $F_{t+1} = \sum_{j=0}^{\infty} \beta^j L_{t+1+j}$ , subject to the IS and aggregate supply equations, taking expectations of future inflation as given. NK model is perfectly forward looking - policy maker recognizes that the choices  $x_t$  and  $\pi_t$  affect the evolution of the problem, but the forward looking model ensures that these do not enter the problem's FOCs. The Lagrange multipliers for the IS and the aggregate supply equations are  $\phi_{1t}$  and  $\phi_{2t}$  respectively. In this case, the implied first order conditions are:

$$\frac{\partial \mathcal{L}}{\partial x_t} : 2\lambda x_t + 2\phi_{1t} - 2\phi_{2t}\kappa' = 0 \quad (18a)$$

$$\frac{\partial \mathcal{L}}{\partial \pi_t} : 2\pi_t + 2\phi_{2t} = 0 \quad (18b)$$

$$i_t \phi_{1t} = 0; i_t \geq 0, \phi_{1t} \geq 0 \quad (18c)$$

We note that for the case where  $\kappa' = \kappa$  the first order conditions are isomorphic to those from CGG (1999). The behavioral parameters  $M_c$  and  $M_f$  do not enter the conditions.

Given the effective lower bound constraint, the central bank's problem can be considered in two phases. Once the natural rate shock has dissipated, the economy enters the lift-off phase in period  $T_d^*$ . Then from  $t = T_d^* + 1$  onwards,  $\phi_{1t} = 0$ . Using the relevant first order conditions, the unique solution is  $\pi_t = 0$ ,  $x_t = 0$  and  $i_t = r_t^n$ .

During the zero-lower bound phase,  $t \leq T_d^*$ ,  $i_t = 0$ . Imposing this condition in the IS and aggregate supply curve, the solution is:

$$A = \begin{bmatrix} \beta M_f + \kappa' \sigma \omega & \kappa' M_c \\ \sigma \omega & M_c \end{bmatrix}, \text{ and } a = \begin{bmatrix} \kappa' \sigma \\ \sigma \end{bmatrix} \quad (19)$$

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = A \begin{bmatrix} E_t \pi_{t+1} \\ E_t x_{t+1} \end{bmatrix} + a r_t^n \quad (20)$$

Since the terminal condition at  $T_d^* + 1$  is:  $[\pi_{T_d^*+1}, x_{T_d^*+1}]' = [0, 0]'$ , we get

$$\begin{bmatrix} \pi_t \\ x_t \end{bmatrix} = \sum_{j=t}^{T_d^*} A^{-(j-t-1)} a r_j^n \quad (21)$$

and  $T_d^*$  is such that  $r_{T_d^*}^n < 0$ , and  $r_{T_d^*+1}^n \geq 0$ . Thus, as for the standard rational expectations case, the dynamics of inflation and output gap are determined by the exogenous natural rate process. The stability of the system will be determined by the eigenvalues of the  $A$  matrix, and these will in turn depend on the behavioral parameters, in addition to the other model parameters.

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