Do central bank announcements and the recent forward guidance provided in these affect the beliefs of market investors? Several analyses have found that a significant proportion of changes in the level of yields in response to these monetary policy statements can be attributed to their effects on investor expectations. For instance, Gürkaynak, Sack and Swanson (2005) find that approximately 75-90% of the change in yields due to monetary policy announcements can be explained due to investors incorporating expectations of the monetary policy actions. Campbell et al. (2012) extend this strategy to find that FOMC announcements had an effect on investor beliefs during the Great Recession. However, Swanson and Williams (2014) find that beginning in late 2011, the sensitivity of yield levels to macroeconomic news (including FOMC announcements) was greatly diminished compared to the earlier period.

Given the above findings, this paper proposes a new method to examine the effect of monetary policy announcements by the FOMC, and its forward guidance in particular, on investor beliefs in the zero-lower bound regime. The method entails extracting moments of the State-Price Densities (SPDs) of investor beliefs about future asset prices using daily data on 2-year and 10-year Futures and the corresponding Option contracts. These moments capture different aspects of investor behavior. The standard deviation of the SPD denotes the ex-ante investor uncertainty about the relevant asset price, the skewness represents the probabilities assigned by investors to positive or negative change in the asset prices and the excess kurtosis captures the weights assigned by investors to "crash" risk. I find that the extracted moments are time-varying, and respond to the issuance of forward guidance. For example, the extension of the zero-lower bound regime, and the announcement of specific purchases about long-term securities are found to lower the probabilities assigned by investors to a tail event. These findings about the higher moments of investor beliefs are economically significant. Time-varying uncertainty about future Treasury yields, which responds to monetary policy announcements, is a channel for affecting the optimizing behavior of investors. In Sinha (2014), I construct a DSGE model with Epstein-Zin preferences and a shock to the interest rate uncertainty is found to generate precautionary savings. Furthermore, incorporating skew and kurtosis in investor preferences is found to affect their portfolio allocation, and the subsequent investment behavior of market participants (Guidolin and Timmermann, 2008).

I. Estimating the State-Price Density of Investor Beliefs

To extract the risk-neutral probability density function associated with U.S. Treasuries, end-of-day daily data on the 2-year and 10-year Future contracts, and the Options written on these are obtained from the Chicago Board of Trade (CBOT) using DataMine (the historical database of the Chicago Mercantile Exchange). These are American-style options, and data is collected for the trading days in 2012 and 2013. The daily data record consists of the following: the type of option (call or put), the month and year of the contract’s expiration, the strike price and the corresponding settlement price, along with other information on the type of trade, volume and implied volatility. Data on these U.S. Futures also reports the month and year at which the Futures contract will be delivered, along with the settlement price of the Futures contract. For each option record, the corresponding Futures price of the appropriate maturity is matched. Using industry convention, the LIBOR rate denominated in U.S. dollars is used as the risk-free rate, for the particular trading day and closest month to maturity.

Following Backus, Foresi and Wu (2004) and

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I would like to thank Linda Goldberg, Linda Tesar, Michael Woodford, Ricardo Reis and participants of the 2014 CeMent workshop for comments and suggestions.
Beber and Brandt (2006), the log futures price $F_t$ over $n$-periods is:

$$\ln F_{t+n} = \ln F_t + \sum_{j=1}^{n} x_{t+j} = \log F_t + x_{t+n}^n.$$  

Given this definition, the conditional distribution of $F_{t+n}$ depends on $x_{t+n}^n$. The valuation of a European-style option on the future with expiration date $t + n$, with strike price $K$ is:

$$C_{t,n,K} = E_t \left[ M_{t,t+n} (F_{t+n} - K)^+ \right],$$

where $M_{t,t+n}$ is the stochastic discount factor and $x^+ \equiv \max (0, x)$. Backus et al. (2004), assume $M$ and $F$ to be independent. Then, with this risk-neutral specification (and assuming market completeness) and $M_{t,t+n} = M (F_t, F_{t+n})$, the call price can be written as:

$$C_{t,n,K} = \int_0^\infty M_{t,t+n} (F_{t+n} - K)^+ p_{t,t+n} dF_{t+n}$$

$$= e^{-r_{nt} n} \int_{\ln(K/F_t)}^\infty (F_{t+n} - K) q_{t,t+n} dF_{t+n}$$

Here $r_{nt}$ is the continuously compounded $n$-period interest rate, $p_{t,t+n} = p (F_t, F_{t+n})$ is the objective probability distribution, $q_{t,t+n} = q (F_t, F_{t+n})$ is the risk-neutral distribution, and the max operator is eliminated by using the bounds of integration.

When the $n$-period log price change is conditionally Gaussian, with standard deviation $\sigma_n$, the risk-neutral distribution of $F_{t+n}$ is conditionally log-normal, and the call price is:

$$C_{t,n,K} = e^{-r_{nt} n} \left[ F_t N (d) - K N (d - \sigma_n) \right],$$

where $d = \frac{\ln(F_t/K) + \sigma_n^2/2}{\sigma_n}$, and $N (x)$ denotes the standard normal cdf at $x$. However, since the SPD can be non-Gaussian, Backus et al. (1997) show that the non-normalities can be captured through a Gram-Charlier expansion of a SPD around a Gaussian density:

$$C_{t,n,K} \approx e^{-r_{nt} n} \left[ F_t N (d) - K N (d - \sigma_n) \right] + F_t e^{-r_{nt} n} \varphi (d) \sigma_n \left[ \frac{\gamma_{1n}}{2!} (2 \sigma_n^2 - d) - \frac{\gamma_{2n}}{3!} (1 - d^2 + 3d \sigma_n - 3 \sigma_n^2) \right]$$

where $\varphi (d)$ is the standard normal density at $d$, and the parameters $\gamma_{1n}$ and $\gamma_{2n}$ correspond to the standard skewness and excess kurtosis respectively.

Finally, the parameters of the expansion of the SPD are estimated by using prices of options with same expiration date, but different strike prices, by numerically solving the nonlinear least-squares (NLLS) problem:

$$\min_{\sigma_n, \gamma_{1n}, \gamma_{2n}} \sum_{i=1}^{N} \left[ C_{t,n,K_i} - C_{t,n,K_i} (\sigma_n, \gamma_{1n}, \gamma_{2n}) \right]^2.$$  

The initial value of the moments used in the NLLS exercise are assumed to be close to the moments for a Gaussian distribution. The main econometric issue in this strategy of extracting the moments of the SPD is that it uses the formulation for European-style options. Melick and Thomas (1997) incorporate the early exercise feature by expressing the American call and put options as convex combinations of upper and lower bounds. The lower bound is the European-style call option price and the upper bound is derived in Chaudhary and Wei (1994). On adopting this approach for the T-bonds for the 1995-1999 period, Beber and Brandt (2006) find that the implied moments are not different from those implied by assuming the European-style call options. Therefore, they conduct their analysis using the European-style option prices. I adopt the same strategy here. Finally, I find the implied moments for only the out-of-the-money options, with $30-90$ days to maturity. The moments of the extracted SPD moments are shown in the online appendix.

\section{II. Responses to FOMC Forward Guidance}

Over 2012 and 2013, the FOMC statements did not announce any changes in the federal funds rate; therefore, unlike the analyses of Gürkaynak et al. (2005) and Goldberg and Grisse (2013), a "surprise" component of monetary policy announcements is not considered here. However, Woodford (2012) and Campbell et al. (2012) emphasize the importance of issuing forward guidance to influence investor expectations in the recent monetary policy regime.

\footnote{Additionally, in the NLLS problem, the problem may imply negative probabilities. Jondeau and Rockinger (2001) derive positivity constraints for the skewness and kurtosis appearing in the Gram-Charlier expansion.}
Therefore, in the empirical strategy, I parse out the forward guidance provided by the Federal Reserve along the following dimensions: (a) an extension of the period for which the zero-lower bound regime would be held in place, as well as the change to threshold-based criterion (instead of the calendar-based guidance); (b) changes in policy regarding purchases of long-term securities; (c) discussion of economic conditions, and whether these were better or worse than expected and (d) the release of the Survey of Economic Projections and accompanying press conferences. In the following specifications, I will consider the effects of these different dimensions of the FOMC announcements on the ex-ante moments of investor beliefs extracted above.

The following econometric specification is used to test whether the extracted moments respond to FOMC statement releases:

$$\Delta m_t = \alpha_t + \beta_t D_{t}^{FOMC} + \theta_t T_t + \epsilon_t.$$ 

Here $\Delta m_t$ represents change in the extracted moments of the SPD; $D_{t}^{A}$ is the dummy variable, with $D_{t}^{FOMC} = 1$ if there is a FOMC meeting, and $T_t$ is the time to maturity. The change in the moments with respect to their levels two days before is considered ($m_t - m_{t-2}$), since there is a general increase in the media coverage before the statement day. Then, to test whether the forward guidance contained in the FOMC statements have an effect, the following dummy variables are also introduced: $D_{t}^{EA}$, which is 1 for the statements that indicated that economic activity had "paused" or "decelerated" (thus indicating worsening economic conditions), $D_{t}^{Dates}$ is 1 if calendar-based guidance is extended, or threshold-based guidance is introduced and $D_{t}^{Long}$ is 1 when specific policies regarding the purchase of long-term securities are announced. The online appendix presents the dates corresponding to these dummy variables. Finally, to examine whether the moments respond to the release of the Survey of Economic Projections (SEP) and press conferences, a dummy variable is introduced, but no statistically significant effects are found.

A. Effects on Standard Deviation

The FOMC statements that extended the zero-lower bound regime, as well as specified the purchases of long-term Treasury securities and Mortgage-Backed Securities (MBSs) are found to increase the ex-ante uncertainty for the 10-year Treasury yield at the 30-90 day horizon, as shown in table 1. These announcements may have been interpreted by investors as the economy doing less well than expected, and the statements were perceived as "negative" news about the economy. The uncertainty corresponding to the 2-year yield is not found to respond to the forward guidance provided in a statistically significant manner.

B. Effects on Skewness

The skewness for the 2-year and 10-year Treasury securities are found to be positive on average. Therefore, during the recovery period of 2012-13, investor beliefs are weighted towards positive (negative) changes in these Treasury yields (prices). Table 2 shows that FOMC announcements have a positive effect on skewness for the 2-year yields (both signed and absolute skewness measures), and but have no effect on the 10-year yield skewness. Statements that indicate specific policy measures regarding the purchases of long-term securities are found to reduce investors' skewness, and push the distribution closer to a Gaussian distribution across the cross-section of maturities. Thus, investors were putting greater probability of no change in either of these Treasury prices following the statements. The extension of the zero-lower bound is also found to reduce the skewness for the 10-year security. The announcements which contain information about worsening economic activity or changes in the expected lift-off dates do not affect the skewness for the 2-year yield.

C. Effects on Excess Kurtosis

For the trading days in 2012 and 2013, the excess kurtosis for the 2-year yield is not statistically different from a Gaussian distribution, while the 10-year distribution shows negative kurtosis with respect to a normal distribution. The latter implies that investors were placing less weight on crash risk, relative to a Gaussian distribution. FOMC announcements are found
to reduce the kurtosis for the 2-year prices, but have no effect on the kurtosis of the 10-year security, as shown in table 3. However, when economic activity is noted to be "paused" or "decelerating", the excess kurtosis rises for the 10-year security, signifying that investors are placing larger mass on the tails of their distribution. The statements regarding further long-term asset purchases and changes in the continuation of the zero-lower bound regime have the opposite effect: in response to these, the excess kurtosis is lowered further for the 10-year yield, implying that investors are reducing their probabilities of a crash risk even more.

III. Conclusions

The analysis above shows that ex-ante investor beliefs respond to forward guidance provided by the FOMC. Even though asset yield levels are not found to respond to the announcements, an exploration of the higher moments shows that market participants are taking the information provided into account. The results further suggest that forward guidance has a greater effect on the investor beliefs about the long-term asset yields. Since these moments affect the investment behavior of economic agents, considering these responses could be significant for the communication of monetary policy.

REFERENCES


Goldberg, L.S. and C. Grisse.: 2013, "Time Variation in Asset Price Responses to Macro Announcements", Federal Reserve Bank of New York Staff Reports No. 626.


### Table 1—Effects on Standard Deviation

<table>
<thead>
<tr>
<th>SPD Mom</th>
<th>ΔStDev 2-year</th>
<th>ΔStDev 10-year</th>
<th>ΔStDev 2-year</th>
<th>ΔStDev 10-year</th>
<th>ΔStDev 2-year</th>
<th>ΔStDev 10-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>-0.05 (1.03)</td>
<td>0.00 (1.11)</td>
<td>-0.05 (1.04)</td>
<td>0.00 (1.05)</td>
<td>-0.05 (1.02)</td>
<td>0.01 (0.30)</td>
</tr>
<tr>
<td>(\beta_{FOMC})</td>
<td>0.04 (0.38)</td>
<td>-0.04 (0.82)</td>
<td>0.03 (0.28)</td>
<td>-0.00 (0.24)</td>
<td>0.06 (0.46)</td>
<td>-0.07* (1.62)</td>
</tr>
<tr>
<td>(\beta_{EA})</td>
<td>0.07 (0.68)</td>
<td>-0.15 (0.92)</td>
<td>-0.09 (0.94)</td>
<td>0.24** (1.76)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_{DATES})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_{LONG})</td>
<td>-0.06 (0.45)</td>
<td>0.19* (1.72)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level using Newey-West standard errors. The t-statistics are shown in the brackets. \(\beta_{FOMC}\) shows the effect of a FOMC announcement, \(\beta_{EA}\) is the effect of a statement with negative news about economic activity, \(\beta_{DATES}\) denotes the change in dates for which the zero-lower bound is expected to be in place and \(\beta_{LONG}\) shows the effect of announcements regarding specific purchases of long-term securities.

### Table 2—Effects on Skewness

<table>
<thead>
<tr>
<th>SPD Mom</th>
<th>ΔSkew 2-year</th>
<th>ΔSkew 10-year</th>
<th>ΔSkew 2-year</th>
<th>ΔSkew 10-year</th>
<th>ΔSkew 2-year</th>
<th>ΔSkew 10-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>-0.04 (0.12)</td>
<td>-0.39* (1.73)</td>
<td>-0.05 (0.15)</td>
<td>-0.39* (1.72)</td>
<td>-0.05 (0.26)</td>
<td>-0.43* (1.88)</td>
</tr>
<tr>
<td>(\beta_{FOMC})</td>
<td>0.48* (1.19)</td>
<td>-0.13 (0.90)</td>
<td>0.45* (1.35)</td>
<td>-0.13 (0.79)</td>
<td>0.35 (1.04)</td>
<td>0.00 (1.87)</td>
</tr>
<tr>
<td>(\beta_{EA})</td>
<td>0.23 (0.69)</td>
<td>-0.01 (0.04)</td>
<td>0.64 (1.01)</td>
<td>-0.04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_{DATES})</td>
<td></td>
<td></td>
<td>-1.04*** (2.55)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_{LONG})</td>
<td></td>
<td></td>
<td>-0.95*** (3.89)</td>
<td>-0.61* (1.42)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level using Newey-West standard errors. The t-statistics are shown in the brackets. \(\beta_{FOMC}\) shows the effect of a FOMC announcement, \(\beta_{EA}\) is the effect of a statement with negative news about economic activity, \(\beta_{DATES}\) denotes the change in dates for which the zero-lower bound is expected to be in place and \(\beta_{LONG}\) shows the effect of announcements regarding specific purchases of long-term securities.

### Table 3—Effects on Kurtosis

<table>
<thead>
<tr>
<th>SPD Mom</th>
<th>ΔExKur 2-year</th>
<th>ΔExKur 10-year</th>
<th>ΔExKur 2-year</th>
<th>ΔExKur 10-year</th>
<th>ΔExKur 2-year</th>
<th>ΔExKur 10-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>0.19 (0.73)</td>
<td>-0.30* (1.43)</td>
<td>0.20 (0.74)</td>
<td>-0.29 (1.36)</td>
<td>0.20 (0.75)</td>
<td>-0.34* (1.56)</td>
</tr>
<tr>
<td>(\beta_{FOMC})</td>
<td>-0.38** (1.86)</td>
<td>0.12 (0.72)</td>
<td>-0.34 (1.54)</td>
<td>0.03 (0.17)</td>
<td>-0.29* (1.38)</td>
<td>0.24* (1.64)</td>
</tr>
<tr>
<td>(\beta_{EA})</td>
<td></td>
<td></td>
<td>-0.41 (1.40)</td>
<td>0.48** (1.86)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_{DATES})</td>
<td></td>
<td></td>
<td>-0.48 (0.87)</td>
<td>-0.91* (1.64)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(\beta_{LONG})</td>
<td></td>
<td></td>
<td>0.50* (1.59)</td>
<td>-0.81*** (2.27)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: * significant at the 10% level, ** significant at the 5% level, *** significant at the 1% level using Newey-West standard errors. The t-statistics are shown in the brackets. \(\beta_{FOMC}\) shows the effect of a FOMC announcement, \(\beta_{EA}\) is the effect of a statement with negative news about economic activity, \(\beta_{DATES}\) denotes the change in dates for which the zero-lower bound is expected to be in place and \(\beta_{LONG}\) shows the effect of announcements regarding specific purchases of long-term securities.