Monetary Policy Uncertainty and Investor Expectations

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Abstract

How does monetary policy uncertainty affect the behavior of market participants? In a New-Keynesian DSGE model with Epstein-Zin preferences, an increase in interest rate uncertainty is found to increase precautionary savings for households, and depress output, inflation and the short- and long-term asset yields. These effects are similar to a negative demand shock. The monetary policy uncertainty shock is calibrated using ex-ante uncertainty of investors about the future changes in Treasury yields, extracted from Options and Futures data. Incorporating the monetary policy uncertainty shock is also found to lower the term premium generated by the model, relative to the case without stochastic volatility in the interest rate rule.

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1 Introduction

> "The pace of business investment has also been only modest during this recovery [...]. Businesses seem not to have had sufficient confidence in the strength and durability of the recovery [...]. Moreover, some analysts have suggested that uncertainty, not only about the strength of the recovery but also about economic policy, could be a significant factor."

Chair Janet Yellen's speech at the Providence Chamber of Commerce, Rhode Island, May 22, 2015.

During the financial crisis, and in its aftermath, policymakers have used several unconventional tools of monetary policy in attempts to influence expectations of market investors. As the level of the Federal Funds Rate fell to the zero-lower bound, the communications and issuances of forward guidance have attempted to influence (and lower) investors' uncertainty about interest rate policy. Uncertainty about interest rates has important implications for decision making by economic agents; the above quote by the Chair of the Federal Reserve recognizes how uncertainty about economic policy could negatively impact investment activity during the recovery period. Changes in uncertainty about interest rates have also been a prevalent feature of the conduct of monetary policy during the mid-2000s. The step-wise increase in the federal funds rate, announced in different statements of the FOMC between June 30, 2004 and June 29, 2006, attempted to guide investor expectations away from the near zero-lower bound in a systematic way. These announcements are hypothesized to have reduced the uncertainty about the path of interest rates. However, even as the Federal Reserve has attempted to influence investor uncertainty about the path of interest rates, the channels through which this uncertainty about monetary policy affects the economy have remained relatively unexplored.

This paper investigates the effects of uncertainty about monetary policy on the optimizing behavior of households. The analysis develops a New-Keynesian model with Epstein-Zin preferences in which monetary policy uncertainty is introduced through a stochastic volatility component in the interest rate rule. Since there are no direct measures of the evolution of uncertainty about monetary policy, the paper uses a novel dataset to do so. Options and Futures data on U.S. Treasuries is used to extract the ex-ante uncertainty about the future change in Treasury yields using the procedure of Beber and Brandt (2006) and Sinha (2015).

Observed prices of Call Options on 30-90 day contracts for 2-year Treasuries, traded on the Chicago Board of Trade, are used to extract the moments of the underlying risk-neutral probability distributions of investors. Empirical studies show that these moments respond to announcements of the Federal Reserve. The standard deviation of the Call Options on these securities are considered as the empirical measure of interest rate uncertainty. In this model, an increase in the interest rate uncertainty is found to encourage precautionary savings for households, and causing a decline in their consumption. Thus, the increased uncertainty about the interest rate acts as a negative demand shock, and depresses inflation as well as short- and long-term interest rates. Finally, the model performance in matching moments on U.S. macroeconomic data is also examined. The analysis thus suggests that if monetary policy can lower the uncertainty about interest rate among investors and households, it may be able to have a positive effect on the output and inflation levels.

The mechanism of the model is similar to the effects of an uncertainty shock to technology or government spending, and has been previously discussed in the literature. However, unlike previous studies, the model here also considers the effects of stochastic volatility in a production economy, and the implications of the uncertainty shock, on the term premia. The simulations indicate that the slope between the 10-year and three-month interest rate falls in response to an increase in the monetary policy uncertainty shock. The analysis also finds that the degree of risk aversion required to generate a positive nominal yield curve is larger than in the model with no stochastic volatility. Intuitively, while holding nominal bonds, the investors face additional consumption uncertainty arising from the stochastic volatitity component of the interest rate rule.

The rest of the paper is organized as follows: a brief review of the literature in section two is followed by a description of the theoretical model in section three. The calibration and simulation of the monetary policy uncertainty shock and subsequent effects on the economy are discussed in section four. Section five concludes.

2 Context in the Literature

This paper is related to the theoretical effects and empirical measurements of policy uncertainty. Mumtaz and Zanetti (2013) use a monetary structural vector autoregression (SVAR), and allow for time-varying variance of monetary policy shocks. This is estimated for U.S.

data, and in response to an increase in monetary policy uncertainty, output, inflation and the nominal interest rate fall. The authors proceed to explain the empirical responses by using a New-Keynesian model based on Ireland (2004) and Sargent and Surico (2011) with stochastic volatility in the interest rate rule. In contrast to this, I use Epstein-Zin preferences to estimate the effects of the monetary policy uncertainty shock on the term premia (among other macroeconomic variables). The calibration of the uncertainty shock is different from the Mumtaz and Zanetti (2013) analysis, and the solution method uses a third-order approximation approach. Hatcher (2011) uses a DSGE model with external habit formation in preferences, and a combination of demand and supply disturbances with time-varying volatilities. The present paper focusses on the effect of time-varying volatilities in the interest rate rule only. Finally, other analyses that examine the effects of uncertainty on the economy are Basu and Bundick (2012) and Leduc and Liu (2014). These consider the effects of a technology and productivity shock respectively. Akkaya (2015) analyzes the effects of changes in the uncertainty about the future path of monetary policy in closed and open economy frameworks. The impact of a change in the volatility of the interest rate is found to be similar to a change in the level of the interest rate. An increase in the uncertainty about the future policy path also leads to an appreciation of the exchange rate. Fernández-Villaverde, Guerrón-Quintana, Kuester, Rubio-Ramírez (2013) examine the effects of changes in uncertainty about future fiscal policy. The authors find that fiscal volatility shocks can generate large adverse effects on the economy, especially at the zero-lower bound. The present analysis considers the effects of a change in uncertainty in the interest rate rule, and the consequent effects on the yield curve and the term premium.

Among the empirical approaches to measuring monetary policy uncertainty shocks, Nakamura and Steinsson (2015) use intra-day data on Fed funds and Eurodollar futures to extract the market expectations about the federal funds rate. They consider the changes in thirty-minute (and one-day windows) around the FOMC announcements, and use these changes as a new measure of monetary shocks. Jurado, Ludvingson and Ng (2015) extract policy uncertainty as the common volatility in the unforecastable component of relevant macroeconomic variables. Baker, Bloom and Davis (2013) develop a measure of general policy uncertainty.

The approach used in this paper attempts to measure monetary policy uncertainty directly from investors in the Futures markets for U.S. Treasuries. This provides a new approach to measuring monetary policy uncertainty for Treasury yields. Since 2-, 5- and 10-

year Treasury Futures and Options are traded, the procedure also develops a new method to estimate investor uncertainty across the term structure of yields.

3 Model Framework

In order to examine the effects of the time-varying nature of conditional volatilities derived from the option-implied state-price densities, and their reactions to the FOMC announcements on the economy, I consider a DSGE model with the following features: households have Epstein-Zin preferences, and choose optimal levels of consumption, one-period bond holdings and labor supply. A final goods sector is perfectly competitive, and uses inputs produced by monopolistically competitive intermediate goods firms. The government makes lump-sum transfers to the households. This follows the setup of Rudebusch and Swanson (2008), Andreasen (2012) and Woodford (2003). The monetary policy authority is assumed to follow a Taylor rule, and responds to the output gap and inflation. In contrast to the standard specifications of the Taylor rule, the monetary policy shock incorporates a stochastic volatility component, in constrast to these papers. In this framework, a sudden change in the uncertainty of the monetary policy shock is considered on the economy. This model allows us to consider the effects of the uncertainty shock in the interest rate rule on the short and long interest rates, as well as the term premium. Unlike the Rudebusch and Swanson (2012) analysis, long-run risks in consumption are not introduced. While these improve the fit of the model to U.S. asset pricing and macroeconomic data, the authors find that a large degree of risk aversion is still required to generate an upward sloping nominal yield curve. Since this long-run uncertainty in inflation may produce interaction effects with the uncertainty in the interest rate rule, it is not considered here. The analysis can then investigate the direct effects of stochastic volatility in the interest rate rule on the term premium.

3.1 Households

The representative household optimally chooses its consumption, c_t, labor supply, l_t, and holdings of the one-period asset B_t. The households maximize the following, subject to the intertemporal budget constraint specified in (4) below.

$$\max E_t \sum_{j=0}^{\infty} \beta^j \frac{\left(c_{t+j}^{\nu}\left(1-l_{t+j}\right)^{1-\nu}\right)^{1-\gamma}}{1-\gamma}. \tag{1}$$

Here $\beta \in (0,1)$ is the discount factor, $\nu \in [0,1]$ and $\gamma \in \mathbb{R}\backslash\{1\}$. Following Rudebusch and Swanson (2012), the value function in (1) is generalized to an Epstein-Zin form in the following way:

$$V_t \equiv \begin{cases} u\left(c_t, l_t\right) + \beta\left(E_t V_{t+1}^{1-\alpha}\right)^{1/(1-\alpha)} & if\ u \geq 0 \text{ everywhere} \\ u\left(c_t, l_t\right) - \beta\left(E_t\left(-V_{t+1}^{1-\alpha}\right)\right)^{1/(1-\alpha)} & if\ u \leq 0 \text{ everywhere} \end{cases}. \tag{2}$$

The utility $u\left(c_t, l_t\right)$ follows the representation in (1), and the parameter $\alpha \in \mathbb{R}$.[2] Given the above specifications, the intertemporal elasticity of substitution is given by $1/\left(1-\nu\left(1-\gamma\right)\right)$, and the relative risk aversion is $\gamma + \alpha\left(1-\gamma\right)$.[3] Using the specific functional form for utility in (1), the household's maximization is represented as:

$$V_t = \frac{\left(c_t\right)^{\nu(1-\gamma)}\left(1-l_t\right)^{(1-\nu)(1-\gamma)}}{(1-\gamma)} - \beta\left[E_t\left(-V_{t+1}\right)^{1-\alpha}\right]^{1/(1-\alpha)}. \tag{3}$$

The household's flow budget constraint is given by:

$$P_t^B B_t + P_t c_t = w_t l_t + d_t + P_t B_{t-1}. \tag{4}$$

Here P_t^B is the price of the one-period asset and w_t is the nominal wage received by households. The price of consumption is P_t, and d_t is the lump-sum transfer received by the households from the firms in period t.

The Euler equations from the household's optimization are given by[4]:

$$-\frac{\partial u\left(c_t, l_t\right)/\partial l_t}{\partial u\left(c_t, l_t\right)/\partial c_t} = \frac{w_t}{P_t}, \tag{5a}$$

$$\frac{\partial u\left(c_t, l_t\right)}{\partial c_t} = \beta E_t\left[\left(E_t V_{t+1}^{1-\alpha}\right)^{\alpha/(1-\alpha)} V_{t+1}^{-\alpha} \frac{\partial u\left(c_{t+1}, l_{t+1}\right)}{\partial c_{t+1}} \frac{P_{t+1}^B}{P_t^B} \frac{P_t}{P_{t+1}}\right]. \tag{5b}$$

[2]If $\alpha = 0$, this representation reduces to the expected utility case with $V \equiv u\left(c_t, l_t\right) + \beta E_t V_{t+1}$.
[3]This is shown by Swanson (2010).
[4]For detailed derivations, please refer to Rudebusch and Swanson (2012).

The nominal stochastic discount factor is:

$$m_{t,t+1} = \left[\frac{V_{t+1}}{\left(E_t V_{t+1}^{1-\alpha}\right)^{\alpha/(1-\alpha)}} \right]^{-\alpha} \beta \frac{\partial u\left(c_{t+1}, l_{t+1}\right)/\partial c_{t+1}}{\partial u\left(c_t, l_t\right)/\partial c_t} \frac{P_t}{P_{t+1}}, \tag{6}$$

and the price of the $n-$period bond $P_t^{n,B}$ is[5]:

$$P_t^{n,B} = E_t \left[m_{t,t+1} P_{t+1}^{n-1,B} \right]. \tag{7}$$

3.2 Firms

There is a continuum of monopolistically competitive intermediate goods firms $i \in [0,1]$, and these produce output using the following production function:

$$y_t^i = z_t a_t \left(\bar{k}^i\right)^\theta \left(l_t^i\right)^{1-\theta}. \tag{8}$$

Here $\bar{k}^i$ is the fixed, firm-specific physical capital stock, l_t^i is the labor input used by firm i, a_t is the aggregate technology, and z_t is the deterministic trend in technology. These firms face quadratic adjustment costs, following Rotemberg (1982). The profits of firm i, given its price p_t^i, and $\xi \geq 0$ are:

$$\max_{l_t^i, p_t^i} \sum_{j=0}^{\infty} m_{t,t+j} \left[\frac{p_{t+j}^i}{P_{t+j}} y_{t+j}^i - w_{t+j} l_{t+j}^i - \frac{\xi}{2} \left(\frac{p_{t+j}^i}{P_{t+j-1}} \frac{1}{\pi_{ss}} - 1 \right)^2 y_{t+j} \right]. \tag{9}$$

The output of firm i is bought by a final goods producer, which is perfectly competitive. The intermediate goods are aggregated to the final product using a CES production function:

$$y_t = \left(\int_0^1 \left[y_t^i\right]^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}, \eta > 1. \tag{10}$$

The demand function faced by firm i for its product is given by:

$$y_t^i = \left(\frac{p_t^i}{P_t} \right)^{-\eta} y_t, \tag{11}$$

[5] This long bond price can be rewritten in terms of the one-period bond price and future stochastic discount factors.

and the aggregate price P_t is determined by the CES aggregate:

$$P_t = \left[\int_0^1 \left[p_t^i\right]^{1-\eta} di\right]^{\frac{1}{\eta-1}}. \tag{12}$$

Finally, technology a_t is assumed to follow the autoregressive process:

$$\ln\left(\frac{a_{t+1}}{a_{ss}}\right) = \rho_a \ln\left(\frac{a_t}{a_{ss}}\right) + \varepsilon_{t+1}^a, \tag{13}$$

where $\varepsilon_{t+1}^a \sim NID\left(0, \sigma_a^2\right)$.

3.3 Policy Makers

There are two policy makers of interest: the central bank and the government.

3.3.1 Central Bank

The central bank is assumed to follow the Taylor rule in setting the nominal short rate:

$$r_t = r_{ss}\left(1-\rho_r\right) + \rho_r r_{t-1} + \phi_\pi \ln\left(\frac{\pi_t}{\pi_{ss}}\right) + \phi_y \ln\left(\frac{y_t}{z_t y_{ss}}\right) + \bar{r}_t. \tag{14}$$

Stochastic volatility is introduced in the the monetary policy shock $\bar{r}_t$, and it evolves according to the process:

$$\bar{r}_{t+1} = \tilde{\rho}_r \bar{r}_t + \sigma_{r,t+1}\vartheta_{t+1}, \tag{15a}$$

$$\ln\left(\frac{\sigma_{r,t+1}}{\sigma_{r,ss}}\right) = \tilde{\rho}_{r,\sigma} \ln\left(\frac{\sigma_{r,t}}{\sigma_{r,ss}}\right) + \varpi_{t+1}. \tag{15b}$$

Here the errors are $\vartheta_{t+1} \sim NID\left(0, \sigma_{\bar{r}}^2\right)$ and $\varpi_{t+1} \sim NID\left(0, \sigma_{\sigma_r}^2\right)$.[6] In the standard model, $\sigma_{r,t+1}$ is assumed to be time-invariant. In the present analysis, an increase in monetary policy uncertainty will affect $\sigma_{r,t+1}$, and subsequently other variables.

[6] Justiniano and Primiceri (2008) hypothesize a similar process for the evolution of an exogenous technology shock.

3.3.2 Government

Government consumption $g_t z_t$ follows the process:

$$\ln\left(\frac{g_{t+1}}{g_{ss}}\right) = \rho_g \ln\left(\frac{g_t}{g_{ss}}\right) + \varepsilon^g_{t+1}. \tag{16}$$

The error is $\varepsilon^g_t \sim NID\left(0, \sigma^2_g\right)$.

3.4 Aggregation

The total output in the economy is:

$$y_t = z_t a_t \left(\bar{k}\right)^{\theta} \left(l_t\right)^{1-\theta}, \tag{17}$$

since all intermediate firms are assumed to be identical. Here $\bar{k}$ denotes the total capital stock and l_t is the labor supply. In equilibrium, the total labor supplied by the households is equal to the labor demanded by the intermediate firms. Finally, the resource constraint is:

$$y_t = c_t + g_t z_t + \delta \bar{k} z_t. \tag{18}$$

Although there is no investment, $\delta \bar{k} z_t$ is used to maintain the fixed capital stock every period.

Finally, the structural relations in the economy are given by equations (3), (5a), (5b), the firm's profit maximization and marginal cost conditions (shown in Appendix 1), (14), (16), (17), along with the evolution of the shocks in (13), (15a), (15b) and (16) the deterministic trend in technology. The state variables are $a_t, g_t, \bar{r}_t, \sigma_{r,t}, \mu_Z$, and the exogenous shocks are $\varepsilon_{a,t}, \varepsilon_{g,t}, \vartheta_{t+1}, \varpi_{t+1}$.

4 Model Solution and Calibration

Since this section focusses on examining the effect of a monetary policy uncertainty shock, the model is solved by a third-order perturbation around the deterministic steady state. Schmitt-Grohe and Uribe (2004) derive the perturbation approximations around the deterministic steady state for a general class of DSGE models, and Andreasen (2012) extends the analysis to the third-order approximation. This methodology accommodates simulating shocks to the volatility of the monetary policy shock, while holding the level of the shock constant.

4.1 Calibration of Model Parameters

The calibration of model parameters is shown in table 1. There are two sets of model parameters that must be set: model parameters, and the parameters of the monetary policy uncertainty shock.

Model parameters

A discussion of the calibration of some other key parameters is useful. Setting $\nu = 0.35$ and $\gamma = 2.5$ yields an intertemporal elasticity of substitution of 0.66. The value of β is set at 0.995, and this implies a low mean value of the three-month nominal interest rate, that is close to the value observed in the data. The implied first and second moments of consumption, inflation, three-month and ten-year nominal interest rates is shown in table 2. These moments are computed from a simulated time series, run for $100,000$ periods, for a third-order approximation of the model. In order to provide comparison with a model with no stochastic volatility, the third column set in table 2 also shows the implied moments for the macroeconomic series with constant volatility for the monetary policy shock, under the third-approximation of the model. The empirical moments of these variables for the period 1972-2014 are also shown.

Among the most notable comparisons between the model with and without stochastic volatility are the increases in the standard deviation of consumption and inflation. In this respect, the findings are similar to those of Rudesbusch and Swanson (2012): the authors find that when long-run inflation risk is included with Epstein-Zin preferences, the volatilities of macro series is higher. Also, in order to generate a positive nominal term premia with stochastic volatility, the required risk aversion parameter is found to be larger in the present analysis. In this case, the households face additional uncertainty while holding on to nominal bonds. Intuitively, when stochastic volatility is introduced in the interest rate rule, it increases future consumption uncertainty for households (relative to the case without stochastic volatility). The added persistence in the volatility component also affects the persistence of future consumption growth. It may be noted that in response to a stochastic volatility shock, we observe a decline in the slope of the yield curve. This is similar to the case without stochastic volatility, and is shown in the impulse responses below. [7].

Uncertainty shock

The U.S. Treasury Futures and Options have been traded on the Chicago Board of Trade (CBOT) since 1977. The initial offerings of the 30-year U.S. Treasury Bond Futures were expanded to include the 10-year, 5-year and 2-year notes by 1990. Debt issued by the U.S. Government is significantly larger in the 2-year maturity sector compared to other maturities, and the CBOT's size of the 2-year Futures contract is larger compared to the other maturities.

The parameters of the monetary policy shock are calibrated using the time-series of standard deviation derived from these U.S. Treasury Futures using the strategy of Beber and Brandt (2006). End-of-day daily data on the 2-year U.S. Futures, and the Options written on these, are obtained from DataMine, the historical database of the CBOT. Data for the trading days in 2012 and 2013 is collected for the American-style options[8]. The daily data record consists of: the type of option (call or put), the month and year of the contract's expiration, the strike price and the corresponding settlement price, along with other information on the type of trade, volume and implied volatility. Data on these U.S. Futures also reports the month and year at which the Futures contract will be delivered, along with the settlement price of the Futures contract. For each option record, the corresponding Futures price of the appropriate maturity is matched. Using industry convention, the LIBOR rate denominated in U.S. dollars is used as the risk-free rate, for the particular trading day and closest month to maturity.

Using the formulations of Backus, Foresi and Wu (2004), the log futures price F_t over

[7] Rudebusch and Swanson (2012) also find that with long-run inflation risk (and Epstein-Zin preferences), the risk aversion required for an upward sloping nominal yield curve is high. The constant relative risk aversion parameter in the best fit of their model is 110.

[8] This is proprietary CBOT data, and access is gained after purchasing the required time series. Therefore, only two years worth of data is used here. Options on longer maturity Treasuries are also traded on the CBOT. However, for the purposes of the present analysis, we will use the shortest maturity available, the 2-year securities.

$n-$periods depends on the change in the futures price, $x_{t+1} = \ln F_{t+1} - \ln F_t$. That is,

$$\ln F_{t+n} = \ln F_t + \sum_{j=1}^{n} x_{t+j} = \log F_t + x_{t+1}^n. \tag{19}$$

This implies that the conditional distribution of F_{t+n} depends on x_{t+1}^n. Furthermore, the valuation of a European-style option on the future with expiration date $t+n$, with strike price K is:

$$C_{t,n,K} = E_t \left[M_{t,t+n} \left(F_{t+n} - K \right)^+ \right], \tag{20}$$

where $M_{t,t+n}$ is the stochastic discount factor and $x^+ \equiv \max(0, x)$. The pricing relation in (20) does not distinguish between the effects of M and F. However, following Backus *et al.* (2004), these are assumed to be independent. Then, with this risk-neutral specification[9], $M_{t,t+n} = M(F_t, F_{t+n})$, and the call price can be written as:

$$\begin{aligned} C_{t,n,K} &= \int_0^{\infty} M(F_t, F_{t+n}) \left(F_{t+n} - K \right)^+ p(F_t, F_{t+n})\, dF_{t+n} \\ &= e^{-r_{nt} n} \int_{\ln(K/F_t)}^{\infty} \left(F_{t+n} - K \right) q(F_t, F_{t+n})\, dF_{t+n} \end{aligned} \tag{21}$$

Here r_{nt} is the continuously compounded $n-$ period interest rate, $q(F_t, F_{t+n})$ is the risk-neutral distribution and the max operator is eliminated by using the bounds of integration.

When the $n-$period log price change is conditionally Gaussian, with mean μ_n and standard deviation σ_n, the risk-neutral distribution of F_{t+n} is conditionally log-normal, and the call price is:

$$C_{t,n,K} = e^{-r_{nt} n} \left[F_t N(d) - K N(d - \sigma_n) \right], \tag{22}$$

where

$$d = \frac{\ln(F_t/K) + \sigma_n^2/2}{\sigma_n}, \tag{23}$$

and $N(x)$ denotes the standard normal cdf at x.[10] However, since the SPD can be non-Gaussian, Backus *et al.* (1997) show that the non-normalities can be captured through a

[9] This also assumes that markets are complete.

[10] As noted by Backus *et al.* (2004), the mean μ_n appears in the formula for d as: $d = \frac{\ln(F_t/K) + \mu_n + \sigma_n^2/2}{\sigma_n}$, and is eliminated from the standard Black-Scholes formula using the no-arbitrage condition. I use this specification to compute the implied conditional mean of the SPD.

Gram-Charlier expansion of a SPD around a Gaussian density:

$$\begin{aligned} C_{t,n,K} &\cong e^{-r_{nt}n}\left[F_t N(d) - KN(d-\sigma_n)\right] \\ &+F_t e^{-r_{nt}n}\varphi(d)\,\sigma_n \begin{bmatrix} \frac{\gamma_{1n}}{3!}(2\sigma_n - d) \\ -\frac{\gamma_{2n}}{4!}(1-d^2+3d\sigma_n-3\sigma_n^2) \end{bmatrix} \end{aligned} \tag{24}$$

where $\varphi(d)$ is the standard normal density at d, and the parameters γ_{1n} and γ_{2n} correspond to the standard skewness and excess kurtosis respectively.

Finally, the parameters of the expansion of the SPD is estimated by using prices of options with same expiration date, but different strike prices, by numerically solving the non-linear least-squares (NLLS) problem:

$$\min_{\mu_n,\sigma_n,\gamma_{1n},\gamma_{2n}} \sum_{i=1}^{N} \left[C_{t,n,K_i} - C_{t,n,K_i}\left(\mu_{n,}\sigma_n,\gamma_{1n},\gamma_{2n}\right)\right]^2. \tag{25}$$

There are several relevant econometric issues with this exercise, and these are discussed in appendix A.2.

The moments of the investors' density function have been used in previous analyses[11] to examine the effect of macroeconomic news announcements, as well as the communications of the Federal Reserve on investor expectations. The standard deviations of the 2-year Option contracts at the 30-90 day windows (upto a quarter) are found to respond to these central bank announcements, and are interpreted as a measure of investors' uncertainty about the interest rate at this time horizon. The stochastic volatility component in the interest rate rule in (15a) is then interpreted as follows: at time t, $\sigma_{r,t}$ represents the ex-ante uncertainty of investors about future change in the interest rate. A sudden increase in this term is interpreted as a rise in uncertainty about the future interest rate. Thus, the moments of the standard deviation series obtained from (25) are used to fit the stochastic volatility component in (15b). Similar to Basu and Bundick (2012),[12] the time-series in (15b) is fit using ordinary least squares to the standard deviation series extracted above. This yields an autocorrelation $\tilde{\rho}_{r,\sigma}$ of 0.82 and σ_{σ_r} of 0.0025.[13]

[11] Examples include Beber and Brandt (2006) and Sinha (2015).

[12] The authors use the VIX series to calibrate the technology shock process.

[13] The daily standard deviation in converted to a quarterly frequency by multiplying with $\sqrt{P}$, where P is the average number of trading days in a quarter.

4.2 Effect of a Monetary Policy Uncertainty Shock

Figure 1 shows the response of the macroeconomic variables of interest to a two standard deviation monetary policy uncertainty shock. The impulse responses are computed as the percent deviation from the ergodic mean of each variable. Following an increase in the volatility of the monetary policy shock, the precautionary savings of households increase, and consumption is lowered. The three-month and the ten-year nominal interest rates also fall. Although the households optimally choose to increase their labor supply, the stickiness in firm's price setting implies lower labor demand following the uncertainty shock. Therefore, in equilibrium, output, consumption and number of hours worked fall. The magnitude of the fall in consumption is 0.14 *b.p.*, and the three-month and ten-year interest rates fall by 0.05 *b.p.* and 0.11 *b.p.* respectively. That is, we observe a decline in the yield curve slope. The magnitude of these changes are similar to the findings of Basu and Bundick (2012), in response to a technology shock. It is may be noted that the response of the ten-year yield to uncertainty shock is in contrast to the findings of Andreasen (2012) for a technology shock. In that analysis, while consumption and the three-month yield fall in response to an increase in the uncertainty about technology, the ten-year yield rises, due to an increase in the quantity of risk.[14]In order to explore the effects of the uncertainty shock on the economy on other dimensions, the following cases are considered: a stronger response of the central bank to inflation, increasing the persistence of the uncertainty shock, and greater price flexibility.

Changing the Central Bank's Response to Inflation The Taylor parameter for inflation, ϕ_π is increased to 2.5, and the other parameters are held fixed at their benchmark values. The effects of a two standard deviation shock in uncertainty on the relevant variables are shown in figure 2. The decline in consumption is more muted, and this is found for the other series as well. Following the initial decline is inflation and labor supply, there is a small increase in the variables, before the return to steady state. As consumption and inflation fall in response to the uncertainty shock, the central bank reacts more aggressively to inflation. Then, as firm markups increase less than in the benchmark case, there is a subsequent rise in the labor supply.

[14] The quantity of risk is derived from the decomposition of the term premia into the price of risk and the quantity.

Changing the Persistence of the Uncertainty Shock In the next simulation, the persistence of the monetary policy uncertainty shock is increased to $\tilde{\rho}_{r,\sigma} = 0.98$. The impulse responses to an increase in the uncertainty shock are shown in figure 3. The decline in the short- and long-term yields persists for significantly longer, and consumption and inflation remain below the steady state for several more periods compared to the benchmark case.

Changing the Price Flexibility Finally, the effect of nominal rigidities on the economy is examined by increasing the degree of price flexibility by approximately 15%. The resulting impulse responses are presented in figure 4. The decrease in consumption is much smaller. Although labor supply falls on impact, it recovers before settling to the steady state. Therefore, the persistence in the decline in consumption is significantly smaller than in the benchmark case. Therefore, nominal rigidities are an important factor in explaining the response of the relevant macroeconomic variables to the monetary policy uncertainty shock.

5 Conclusion

The analysis in this paper investigates the effects of changes in the uncertainty on the economy. A rise in uncertainty is found to have a recessionary impact on the economy. Data on U.S. Treasury Futures and Options is used to calibrate the monetary policy uncertainty shock, and in response to a rise in uncertainty, the yield curve slope declines. The empirical strategy for calibrating the uncertainty shock also provides a means of exploring the quantitative effects of forward guidance on the economy: if the ex-ante uncertainty of investors about the change in future yields reacts to statements of the Federal Reserve, then the size of this effect can be used to calibrate a "forward guidance shock", in which the uncertainty responds by the magnitude found in the data. The analysis also finds that the effects of this interest rate uncertainty on macroeconomic variables are quantitatively small. Therefore, one avenue for further research is to investigate if shocks to uncertainty have larger effects if the economy is at the zero-lower bound.

Appendix 1

The structural relations of the DSGE model are:

1. The household's maximization problem:

$$\tilde{V}_t = \frac{\left(\tilde{C}_t\right)^{\nu(1-\gamma)} (1-l_t)^{(1-\nu)(1-\gamma)}}{(1-\gamma)} - \beta \left[E_t \left(-\tilde{V}_{t+1} \mu_{Z,t+1}^{\nu(1-\gamma)} \right)^{1-\alpha} \right]^{1/(1-\alpha)}$$

2. The labor supply condition:

$$\frac{(1-\nu)\,\tilde{C}_t}{\nu\,(1-l_t)} = \tilde{w}_t$$

3. The Euler equation:

$$P_t^B = E_t \left[\beta \left(\frac{\left(E_t \left[-\tilde{V}_{t+1} \right]^{1-\alpha} \right)^{1/(1-\alpha)}}{-\tilde{V}_{t+1}} \right)^a \mu_{Z,t+1}^{\nu(1-\gamma)-1} \frac{\left(\tilde{C}_{t+1}\right)^{\nu(1-\gamma)-1} (1-l_{t+1})^{(1-\nu)(1-\gamma)}}{\left(\tilde{C}_t\right)^{\nu(1-\gamma)-1} (1-l_t)^{(1-\nu)(1-\gamma)}} \frac{P_t}{P_{t+1}} \right]$$

4. The firm's marginal cost condition:

$$mc_t = \frac{\tilde{w}_t}{(1-\theta)\,A_t \bar{k}^{\theta} l_t^{-\theta}}$$

5. Firm's optimality:

$$\begin{aligned} mc_t &= \frac{\eta-1}{\eta} - E_t \left[\begin{array}{c} \beta \left(\frac{\left(E_t \left[-\tilde{V}_{t+1} \right]^{1-\alpha} \right)^{1/(1-\alpha)}}{-\tilde{V}_{t+1}} \right)^a \mu_{Z,t+1}^{\nu(1-\gamma)-1} \\ \frac{\left(\tilde{C}_{t+1}\right)^{\nu(1-\gamma)-1} (1-l_{t+1})^{(1-\nu)(1-\gamma)}}{\left(\tilde{C}_t\right)^{\nu(1-\gamma)-1} (1-l_t)^{(1-\nu)(1-\gamma)}} \frac{\xi}{\eta} \left(\frac{\pi_{t+1}}{\pi_{ss}} - 1 \right) \frac{\pi_{t+1} \tilde{Y}_{t+1}}{\pi_{ss} \tilde{Y}_t} \end{array} \right] \\ &\quad + \frac{\xi}{\eta} \left(\frac{\pi_t}{\pi_{ss}} - 1 \right) \frac{\pi_t}{\pi_{ss}} \end{aligned}$$

6. The Taylor rule:

$$r_t = r_{ss}(1-\rho_r) + \rho_r r_{t-1} + \phi_\pi \ln\left(\frac{\pi_t}{\pi_{ss}}\right) + \phi_y \ln\left(\frac{y_t}{z_t y_{ss}}\right) + \bar{r}_t$$

7. The production function:

$$y_t = z_t a_t \left(\bar{k}\right)^\theta (l_t)^{1-\theta}$$

8. Resource constraint:

$$y_t = c_t + g_t z_t + \delta \bar{k} z_t$$

9. Technology:

$$\ln\left(\frac{a_{t+1}}{a_{ss}}\right) = \rho_a \ln\left(\frac{a_t}{a_{ss}}\right) + \varepsilon_{a,t+1}$$

10. Government:

$$\ln\left(\frac{g_{t+1}}{g_{ss}}\right) = \rho_g \ln\left(\frac{g_t}{g_{ss}}\right) + \varepsilon^g_{t+1}$$

11. Monetary policy shock: $\bar{r}_{t+1}$For the approximation, this is represented as:

$$\begin{aligned} \bar{r}_{t+1} &= \tilde{\rho}_r \bar{r}_t + \sigma_{r,t+1}\vartheta_{t+1}, \\ \ln\left(\frac{\sigma_{r,t+1}}{\sigma_{r,ss}}\right) &= \tilde{\rho}_{r,\sigma} \ln\left(\frac{\sigma_{r,t}}{\sigma_{r,ss}}\right) + \varpi_{t+1}. \end{aligned}$$

12. Deterministic trend:

$$\begin{aligned} \ln\left(\mu_{Z,t+1}\right) &= \ln\left(\mu_{Z,ss}\right) \\ \mu_{Z,t+1} &= \frac{Z_{t+1}}{Z_t} \end{aligned}$$

Variables that are transformed in the above relations: $\tilde{C}_t = \frac{C_t}{Z_t}, \tilde{w}_t = \frac{w_t}{Z_t}, \tilde{Y}_t = \frac{Y_t}{Z_t}, \tilde{V}_t = \frac{V_t^{\nu(1-\gamma)}}{Z_t}$.

Appendix 2

In the NLLS exercise in (25), the initial value of the moments used in the NLLS exercise are assumed to be close to the moments for a Gaussian distribution. The main econometric issue in this strategy of extracting the moments of the SPD is that it uses the formulation for European-style options. Melick and Thomas (1997) incorporate the early exercise feature by expressing the American call and put options as convex combinations of upper and lower bounds. The lower bound is the European-style call option price and the upper bound is derived in Chaudhary and Wei (1994). On adopting this approach for the T-bonds for the 1995-1999 period, Beber and Brandt (2006) find that the implied moments are not different from those implied by assuming the European-style call options. Therefore, they conduct their analysis using the European-style option prices. I adopt the same strategy here. Finally, I find the implied moments for only the out-of-the-money options, with $30 - 90$ days to maturity.[15] The moments of the extracted SPD moments are shown in the online appendix.

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Tables and Figures

Table 1: Model Parameters

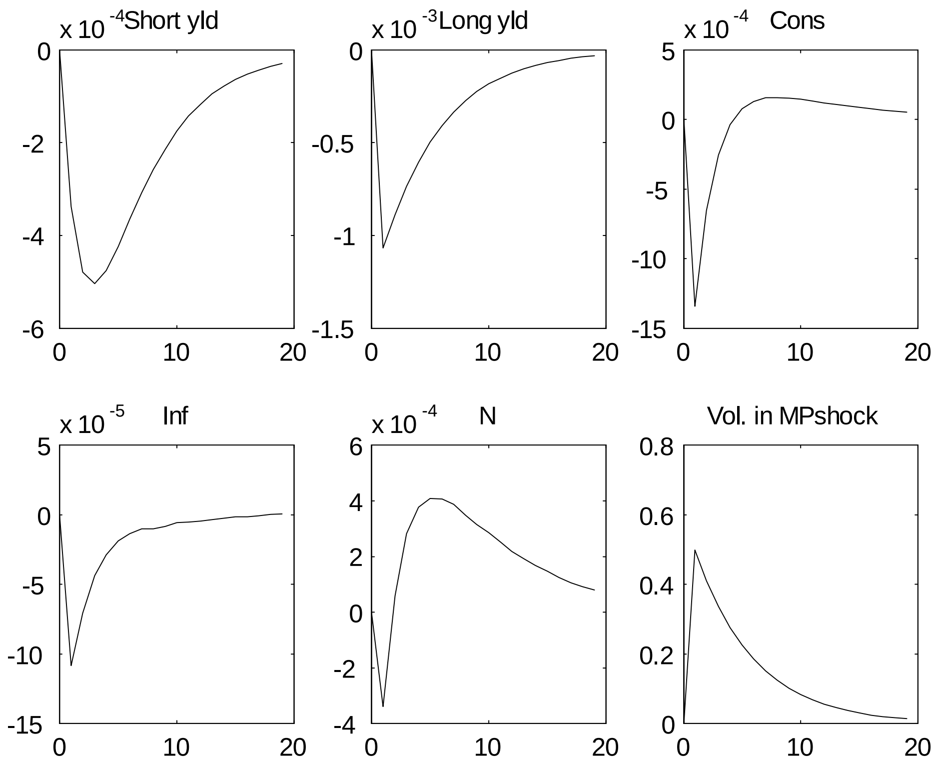
Coeff	Value	Coeff	Value	Coeff	Value	Coeff	Value
γ	2.5	ξ	260	$\frac{g_{ss}}{y_{ss}}$	0.17	$\text{std}(\varepsilon_{a,t})$	0.0075
ν	0.35	δ	0.025	π_{ss}	1.008	$\text{std}(\varepsilon_{g,t})$	0.004
β	0.995	ϕ_{π}	1.5	$\mu_{z,ss}$	1.005	$\text{std}(\vartheta_t)$	0.003
α	-148	ϕ_y	0.3	ρ_a	0.98	$\text{std}(\varpi_t)$	0.15
θ	0.36	ρ_r	0.85	ρ_g	0.90	$\tilde{\rho}_{r,\sigma}$	0.92
η	6	l_{ss}	0.38	$\tilde{\rho}_r$	0.90		

Table 2: Implied Moments of Macroeconomic Series

Moment / Macroeconomic Variable	U.S. Data (1972-2007)		With SV		Without SV	
	Mean	Sd.	Mean	Sd.	Mean	Sd.
Consumption	4.04	2.65	1.99	6.32	1.99	4.09
Inflation	3.94	2.51	3.17	6.40	3.21	2.89
$3 - month\ yield$	5.94	2.90	6.19	3.92	5.44	3.03
$10 - year\ yield$	7.55	2.39	5.67	1.21	6.75	2.08

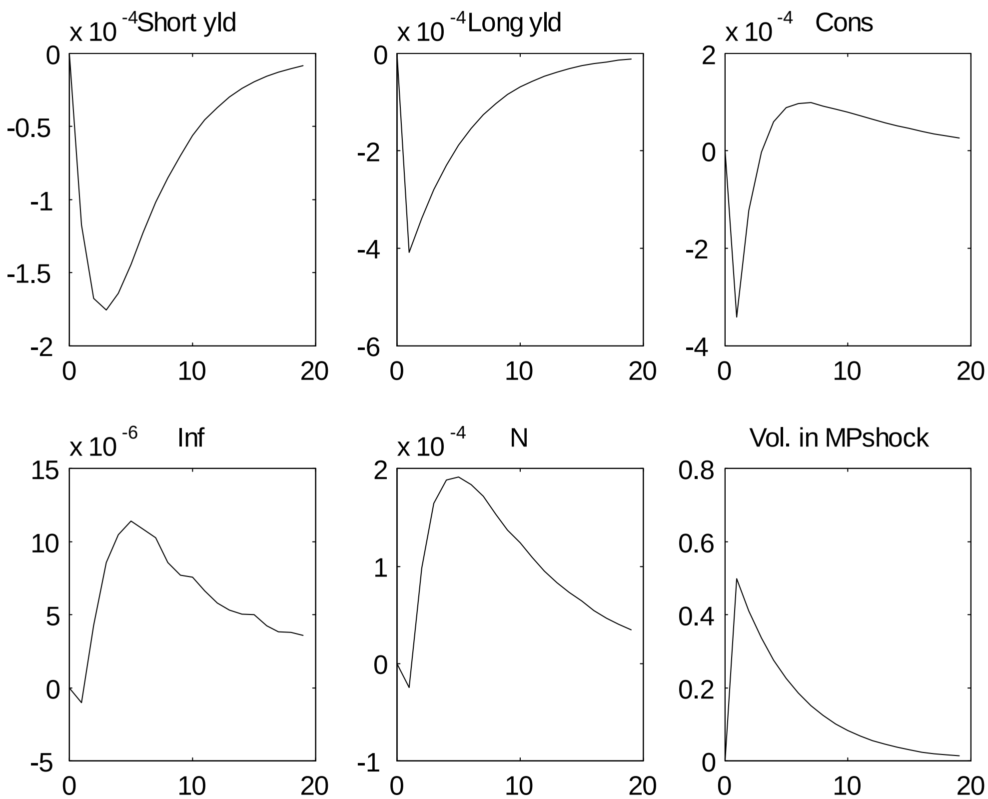
Note: For U.S. data, consumption growth is based on Real Personal Consumption Expenditure for non-durables and services, inflation is based on the GDP Implicit Price Deflator, 3-month yield is the 3-month Treasury Bill rate (secondary market) and the 10-year yield is obtained from the Gurkaynak, Sack and Wright (2007) fitted yield curve. All variables are quarterly values expressed in percent. Inflation and interest rates are expressed at annual rates.

Figure 1: Impulse Response to a Monetary Policy Uncertainty Shock
(Benchmark)



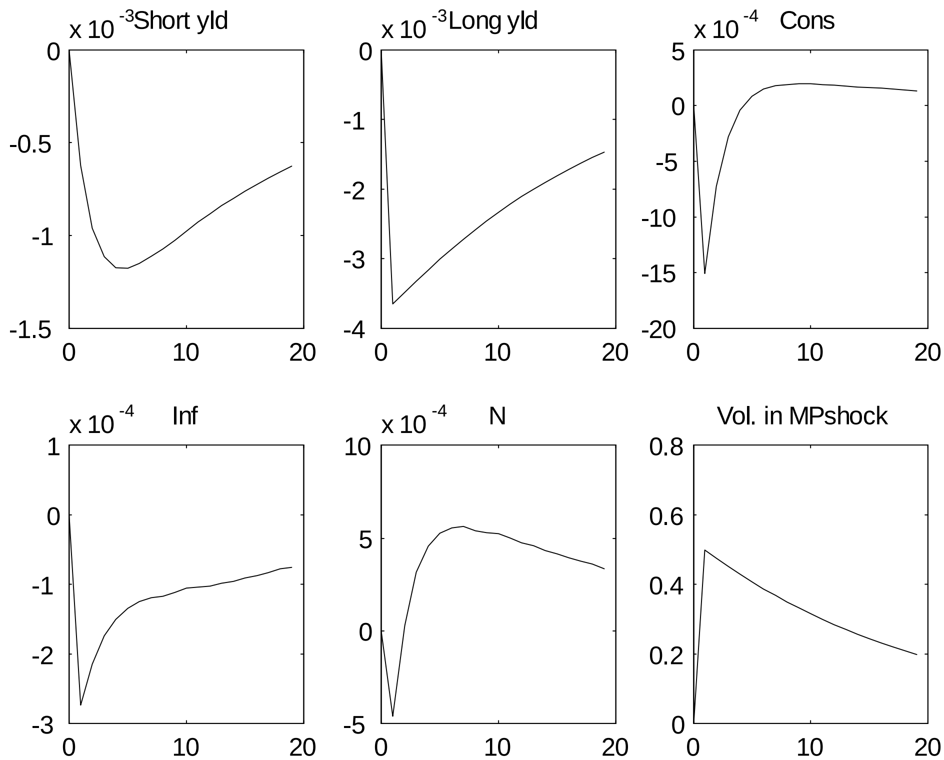
Note: These are percentage deviations from the ergodic mean in response to a two standard deviation shock to monetary policy uncertainty. The parameters are fixed at their benchmark values. The variables shown are the 3-month interest rate (Short Yld), the ten-year interest rate (Long Yld), consumption (Cons), inflation (Inf), labor supply (N) and the volatility of the MP shock (Vol. in MPshock).

Figure 2: Impulse Response to a Monetary Policy Uncertainty Shock
(More Aggresive Response to Inflation by the Central Bank)



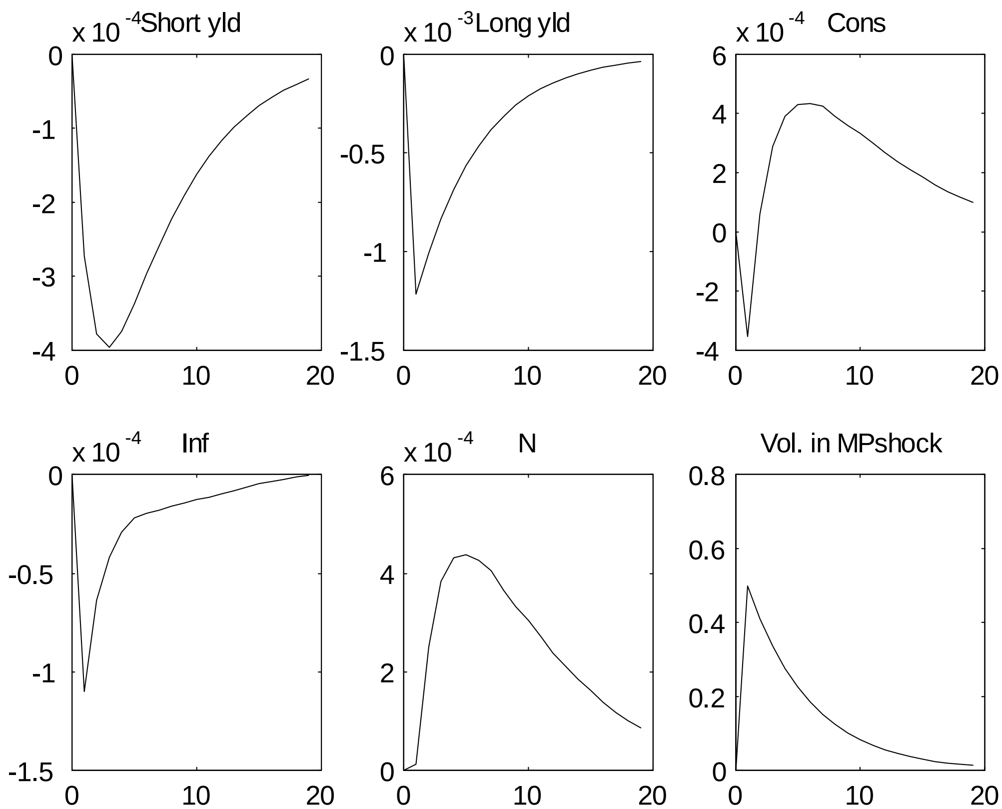
Note: These are percentage deviations from the ergodic mean in response to a two standard deviation shock to monetary policy uncertainty. The Taylor rule parameter ϕ_π is increased to 2.5; the remaining parameters are fixed at their benchmark values. The variables shown are the 3-month interest rate (Short Yld), the ten-year interest rate (Long Yld), consumption (Cons), inflation (Inf), labor supply (N) and the volatility of the MP shock (Vol. in MPshock).

Figure 3: Impulse Response to a Monetary Policy Uncertainty Shock
(Greater Persistence in the Monetary Policy Uncertainty Shock)



Note: These are percentage deviations from the ergodic mean in response to a two standard deviation shock to monetary policy uncertainty. The persistence parameter $\tilde{\rho}_{r,\sigma}$ is increased to 0.98; the remaining parameters are fixed at their benchmark values. The variables shown are the 3-month interest rate (Short Yld), the ten-year interest rate (Long Yld), consumption (Cons), inflation (Inf), labor supply (N) and the volatility of the MP shock (Vol. in MPshock).

Figure 4: Impulse Response to a Monetary Policy Uncertainty Shock
(Greater Price Flexibility)



Note: These are percentage deviations from the ergodic mean in response to a two standard deviation shock to monetary policy uncertainty. The price flexibility is increased by 15%; the remaining parameters are fixed at their benchmark values. The variables shown are the 3-month interest rate (Short Yld), the ten-year interest rate (Long Yld), consumption (Cons), inflation (Inf), labor supply (N) and the volatility of the MP shock (Vol. in MPshock).