A Mathematical Appendix

Appendix A.1. Intertemporal Optimization of Agents:

The first order approximation of the optimality conditions of the households and firms is constructed around the following steady state: $\xi = 0$, $Y_t = \bar{Y}$ (defined below) and $\bar{P}_1^B = \beta$ (or $\bar{\imath}_1 = (1-\beta)/\beta$) with $\bar{\pi} = 1$. The hat variables denote the log deviations of the respective variable from its steady state value. For the one-period interest rate, the log deviation is defined as $\hat{\imath}_{1,t} = \log[(1+i_{1,t})/(1+\bar{\imath}_1)]$. The first order approximation of the household's Euler equation for the one-period asset price yields:

$$\hat{C}_t^i = \tilde{E}_t \hat{C}_{t+1}^i - \sigma(\hat{\imath}_{1,t} - \hat{\pi}_{t+1}) + (g_t - \tilde{E}_t g_{t+1}). \tag{1}$$

The flow budget constraint in (4) is iterated forwards, and its approximation is:

$$\tilde{E}_t \sum\nolimits_{j=0}^{\infty} \beta^j \hat{C}_{t+j}^i = \hat{W}_t^i + \tilde{E}_t \sum\nolimits_{j=0}^{\infty} \beta^j \hat{Y}_{t+j}^i. \tag{2}$$

Substituting (1) recursively into (2) yields:

$$\hat{C}_t^i = (1-\beta)\hat{W}_t^i + (1-\beta)\tilde{E}_t \sum\nolimits_{j=0}^{\infty} \beta^j \left[\hat{Y}_{t+j}^i - \sigma\beta(\hat{\imath}_{1,t+j} - \hat{\pi}_{t+j+1}) + \beta(g_{t+j} - g_{t+j+1})\right].$$

The output gap is defined as $\hat{x}_t = \log(Y_t/Y_t^n)$. Aggregating across i households, and applying market clearing conditions yields:

$$\hat{x}_t = \tilde{E}_t \sum\nolimits_{j=0}^{\infty} \beta^j \left[(1-\beta)\hat{x}_{t+j+1} - \sigma\beta(\hat{\imath}_{1,t+j} - \tilde{E}_t\hat{\pi}_{t+j+1}) + \hat{r}_{t+j+1}^n\right], \tag{3}$$

with the natural rate of interest $\hat{r}_t^n = (\hat{Y}_{t+1}^n - g_{t+1}) - (\hat{Y}_t^n - g_t)$.

Before deriving the approximation to the firm's optimization problem, the real marginal cost function is defined using $s_{t,t+j}$ as firm k'smarginal cost in period $t+j$:

$$s(y, Y, \bar{\xi}) = \frac{v_h(f^{-1}(y/A;\xi)}{u_c(Y,\xi)A}\frac{1}{f'(f^{-1}(y))}, \tag{4}$$

where $\bar{\xi} \equiv (\xi, A)$ is a vector of preference and technology shocks. When prices are fully flexible, the price of firm k is a markup over its real marginal cost, $\frac{p_t(k)}{P_t} = \mu s(y_t(k), Y_t, \bar{\xi}_t)$,where $\mu = \theta/(\theta-1)$.Then, in equilibrium, the firms will face the symmetric problem, so that the price set by each firm k is P_t and its output is Y_t. This implies that $s(Y_t^n, Y_t^n, \bar{\xi}_t) = \mu^{-1}$. The natural rate of output Y_t^n is thus defined. This relation is also used to define the steady state level of output $\bar{Y}$ such that $s(\bar{Y}, \bar{Y}, 0) = \mu^{-1}$.

The linearization of (4) gives $\hat{s}_{t,t+j}(k) = \omega\hat{y}_{t+j}(k) + \sigma^{-1}\hat{Y}_{t+j} - (\omega + \sigma^{-1})\hat{Y}_{t+j}^n$,where $\omega > 0$ is the elasticity of the real marginal cost function $s(.)$ with respect to $y_t(k)$. Aggregating this relation yields $\hat{s}_{t+j} = (\omega + \sigma^{-1})(\hat{Y}_{t+j} - \hat{Y}_{t+j}^n)$.This implies the following relation between the real marginal cost of producing $y_t(k)$ and the aggregate output Y_t : $\hat{s}_{t,t+j}(k) = \hat{s}_{t+j} - \omega\theta\left[\hat{p}_t(k) - \sum_{m=t+1}^{t+j}\hat{\pi}_m\right]$. Finally, to derive the Phillips curve, differentiate the firm's optimization problem with respect to $p_t(k)$, and use this relation to get:

$$\hat{p}_t^* = \tilde{E}_t \sum\nolimits_{j=0}^{\infty} (\alpha\beta)^j \left[\frac{1-\alpha\beta}{1+\omega\theta}(\omega + \sigma^{-1})\hat{x}_{t+j} + \hat{\pi}_{t+j}\right]. \tag{5}$$

The can be rewritten using the approximation to the aggregate price index: $\hat{\pi}_t = \hat{p}_t^*(1-\alpha)/\alpha$.

Appendix A.2. Proof of Proposition 1:

In the flexible price limit of the benchmark model considered here, the one-period asset price is only a function of the exogenous endowment process: $\hat{P}_{1,t} = a_t + b_t\hat{y}_{t-1} + \eta_t$.With rational expectations, the fixed points of the beliefs are $\bar{a} = 0$ and $\bar{b} = \rho(1-\rho)/\sigma$, where ρ is the AR(1) parameter of the endowment process. The only process that will be forecasted by optimizing households is $\hat{P}_{1,t}$, since $\hat{y}_t$ is assumed to be known with probability one. The actual evolution of the one period asset price will be determined as $\hat{P}_{1,t} = T_0^p(a_t, b_t) + T_z^p(a_t, b_t)\hat{y}_{t-1} + \eta_t$. Now, the $T-$ mappings are constructed as:

$$T^{0,p}(a_t, b_t) = \frac{-\beta}{1-\beta} a_t^p \tag{6}$$

$$T^{z,p}(a_t, b_t) = \rho\left[\frac{(1-\beta)\rho}{\sigma(1-\beta\rho)} - \frac{\beta b_t^{p,y}}{1-\beta\rho}\right] + \frac{\rho}{\sigma} \tag{7}$$

The rational expectations equilibrium (REE) is defined as a fixed point of the mappings in (6) and (7): $T(\bar{a}_t, \bar{b}_t) = (\bar{a}_t, \bar{b}_t)$. Evans and Honkapohja (2001) use stochastic approximation results to show that the learning algorithm in (17)[1] converges to this REE if the following ordinary differential equation is locally stable: $\frac{\partial}{\partial \tau}(a_t, b_t) = T(a_t, b_t) - (a_t, b_t)$.The required Jacobian, evaluated at $(\bar{a}_t, \bar{b}_t)$ is $J(\bar{a}_t, \bar{b}_t) = \begin{bmatrix} -\frac{\beta}{1-\beta} & 0 \\ 0 & -\frac{\beta\rho}{1-\beta\rho} \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

As $J(\bar{a}_t, \bar{b}_t)$ has negative eigenvalues for $0 < \beta, \rho < 1$, the conditions for E-stability are satisfied. To obtain the asymptotic distribution of the parameters, I consider the system where the intercept term is not estimated, and set $S_{t-1} = R_t$ to get:

$$\begin{aligned} b_t &= b_{t-1} + gS_{t-1}^{-1}\hat{y}_{t-1}[db_{t-1}\hat{y}_{t-1} + V_{b,t-1}\varepsilon_t] \\ S_t &= S_{t-1} + g(\hat{y}_t^2 - S_{t-1}) \end{aligned} \tag{8}$$

This is in the standard form: $\theta_t = \theta_{t-1} + g\mathcal{H}(\theta_{t-1}, X_t)$ where

$$\theta_t = \begin{pmatrix} b_t \\ S_t \end{pmatrix}, \mathcal{H}(\theta_{t-1}, X_t) = \begin{pmatrix} \mathcal{H}_b(\theta_{t-1}, X_t) \\ \mathcal{H}_S(\theta_{t-1}, X_t) \end{pmatrix}, X_t = \begin{pmatrix} b_t \\ b_{t-1} \\ \varepsilon_t \end{pmatrix} \tag{9}$$

and $\mathcal{H}_b(\theta_{t-1}, X_t) = S_{t-1}^{-1}\hat{y}_{t-1}[db_{t-1}\hat{y}_{t-1} + V_{b,t-1}\varepsilon_t]$, $\mathcal{H}_S(\theta_{t-1}, X_t) = (\hat{y}_t^2 - S_{t-1})$.

For infinite horizon asymptotic results, by Theorem 7.9 of Evans and Honkapohja (2001), the distribution of θ_t can be approximated for small g and large t as $\theta_t \sim N(\theta^{RE}, gC)$ where $\theta^{RE} = (\rho(1-\rho), E\hat{y}_t^2)'$, and $C = \int_0^\infty e^{sB}\mathcal{R}^* e^{sB'} ds$. Then,

$$\begin{aligned} h_b(b, S) &= \lim_{t\longrightarrow\infty} E\mathcal{H}_b(\theta_{t-1}, X_t) = S^{-1}E\hat{y}_t^2\left[\frac{\rho}{\sigma} + \frac{\rho^2(\beta-1)}{\sigma(1-\beta\rho)} + b\left(\frac{-\beta\rho}{1-\beta\rho} - 1\right)\right] \\ h_S(b, S) &= \lim_{t\longrightarrow\infty} E\mathcal{H}_S(\theta_{t-1}, X_t) = E\hat{y}_t^2 - S \end{aligned} \tag{10}$$

Also,

$$B = D_\theta h(\theta^{RE}) = \begin{pmatrix} \left(\frac{-1}{1-\beta\rho}\right) & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} b_{11} & 0 \\ 0 & b_{22} \end{pmatrix} \tag{11}$$

[1] Equation reference in original paper.

$$\mathcal{R}^{ij}(\theta)=\sum_{k=-\infty}^{\infty} cov[\mathcal{H}^i(\theta, X_k^\theta), \mathcal{H}^j(\theta, X_0^\theta)]$$

Considering only b, $\mathcal{H}_b(\theta, X^\theta) = S_{t-1}^{-1}\hat{y}_{t-1}[db\hat{y}_{t-1}+V_b\varepsilon_t]$ and $\mathcal{R}^{ij}(b) = \sum_{k=-\infty}^{\infty}[...+cov[\mathcal{H}^i(\theta, X_0^\theta), \mathcal{H}^j(\theta, X_0^\theta)]+cov[\mathcal{H}^i(\theta, X_1^\theta), \mathcal{H}^j(\theta, X_0^\theta)] + ...]$. At θ^{RE}, $cov[\mathcal{H}^i(\theta, X_t^\theta), \mathcal{H}^j(\theta, X_t^\theta)] = \frac{(1-\rho)^2}{\sigma^2}\frac{\sigma_\varepsilon^2}{E\hat{y}_t^2}$. This is valid since the unconditional expectation is taken as $t \longrightarrow \infty$. Since $\hat{y}_{t-1}^2, \varepsilon_t^2$ are independent random variables, $E(\hat{y}_{t-1}^2\varepsilon_t^2) = E\hat{y}_{t-1}^2\sigma_\varepsilon^2$. Other sums in the series are $cov[\mathcal{H}^i(\theta, X_{t+1}^\theta), \mathcal{H}^j(\theta, X_t)] = \frac{1}{(E\hat{y}_t^2)^2}\frac{(1-\rho)^2}{\sigma^2}E(\hat{y}_t\varepsilon_{t+1}\hat{y}_{t-1}\varepsilon_t) = 0$. Therefore, for b, $\mathcal{R}^*(b) = \frac{(1-\rho)^2}{\sigma^2}\frac{\sigma_\varepsilon^2}{E\hat{y}_t^2}$and $C = \frac{(1-\rho)^2}{\sigma^2}\frac{(1-\beta\rho)(1-\rho^2)}{2}$.

Appendix A.3. Proof of Proposition 2:

I first consider the Expectations Hypothesis regression for $n = 2$:

$$\hat{\imath}_{1,t+1} - \hat{\imath}_{2,t} = \alpha + \gamma(\hat{\imath}_{2,t} - \hat{\imath}_{1,t}) + e_t \tag{12}$$

where $\hat{\imath}_{1,t} = -T_{b,t-1}\hat{y}_{t-1} - V_{b,t-1}\varepsilon_t$ and $\hat{\imath}_{1,t+1} = -T_{b,t}\hat{y}_t - V_{b,t}\varepsilon_{t+1}$, $\hat{\imath}_{2,t} = \frac{1}{2}(y_{1,t} + E_t y_{1,t+1})$.

In the regression in (12), define X and Y where

$$\begin{aligned} X &= \hat{\imath}_{2,t} - \hat{\imath}_{1,t} = \frac{\hat{y}_{t-1}}{2}T_{b,t-1}(1-\rho) - \frac{\varepsilon_t}{2}(T_{b,t-1} - V_{b,t-1}) \qquad (13) \\ Y &= \hat{\imath}_{1,t+1} - \hat{\imath}_{2,t} = \hat{y}_{t-1}\left[-T_{b,t}\rho + \frac{1}{2}T_{b,t-1}(1+\rho)\right] + \varepsilon_t(-T_{b,t} + \frac{1}{2}(V_{b,t-1} + T_{b,t-1})) - V_{b,t}\varepsilon_{t+1} \end{aligned}$$

For computing the asymptotic bias in γ:

$$bias = \frac{\begin{array}{c} p\lim_{T\longrightarrow\infty} T^{-1}\sum_{t=1}^{T}(X_t[\varepsilon_t(-T_{b,t} + \frac{1}{2}(V_{b,t-1} + T_{b,t-1})) - V_{b,t}\varepsilon_{t+1}]) \\ -p\lim_{T\longrightarrow\infty}\left(T^{-1}\sum_{t=1}^{T} X_t\right) p\lim_{T\longrightarrow\infty}\left(T^{-1}\sum_{t=1}^{T} e_t\right) \end{array}}{p\lim_{T\longrightarrow\infty}(T^{-1}\sum_{t=1}^{T} X_t^2 - (T^{-1}\sum_{t=1}^{T} X_t)^2)} \tag{14}$$

Consider the following term:

$$T^{-1}\sum_{t=1}^{T} X_t = T^{-1}\sum_{t=1}^{T}\left[\frac{1}{2}T_{b,t-1}(1-\rho)\hat{y}_{t-1} - \frac{1}{2}\varepsilon_t(T_{b,t-1} - V_{b,t-1})\right] \tag{15}$$

From Evans and Honkapohja (2001), when the E-stability conditions are satisfied (as they are for the present case), for the constant gain algorithm, the following results are obtained: (a) the estimates of the updated coefficients are unbiased asymptotically, i.e., $E(b_t) \longrightarrow b^{RE}$ as $t \longrightarrow \infty$; (b) b_t approaches a limiting normal distribution $N(b^{RE}, gC)$ where C is the variance-covariance matrix of the updated coefficients.

From adaptive learning $b_t = b_{t-1} + gR_t^{-1}\hat{y}_{t-1}[(T_{b,t-1} - b_{t-1})\hat{y}_{t-1} + V_{b,t-1}\varepsilon_t]$. I further assume that R_t is not updated, and is equal to the RE value: $\bar{R}$. The relevant T-mappings are:

$$\begin{aligned} T_{b,t} &= \frac{\rho}{\sigma} + \rho\left[\frac{-\beta b_t}{1-\beta\rho} + \frac{\rho(\beta-1)}{\sigma(1-\beta\rho)}\right] \qquad (16) \\ V_{b,t} &= \left[\frac{-\beta b_t}{1-\beta\rho} + \frac{\rho(\beta-1)}{\sigma(1-\beta\rho)}\right] + \frac{1}{\sigma} \end{aligned}$$

Then,

$$p \lim_{T \longrightarrow \infty} T^{-1} \sum_{t=1}^{T} (T_{b,t-1} \hat{y}_{t-1}) = -\frac{\beta\rho}{1-\beta\rho} p \lim_{T \longrightarrow \infty} T^{-1} \sum_{t=1}^{T} \left(b_{t-1} \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-1-j} \right) \tag{17}$$

Using the independence of random variables, and the weak law of large numbers, $T^{-1} \sum_{t=1}^{T} (T_{b,t-1} \hat{y}_{t-1}) = 0$. For the second term in $T^{-1} \sum_{t=1}^{T} (X_t)$:

$$T^{-1} \sum_{t=1}^{T} [\varepsilon_t (T_{b,t-1} - V_{b,t-1})] = (1-\rho) \frac{\beta}{1-\beta\rho} T^{-1} \sum_{t=1}^{T} (\varepsilon_t b_{t-1}) \tag{18}$$

Rewriting $T^{-1} \sum_{t=1}^{T} (\varepsilon_t b_{t-1})$, and applying $p \lim_{T \longrightarrow \infty} T^{-1} \sum_{t=1}^{T} (\varepsilon_t b_{t-1}) = p \lim_{T \longrightarrow \infty} T^{-1} \sum_{t=1}^{T} (X_t) = 0$, we get:

$$\begin{aligned} & T^{-1} \sum_{t=1}^{T} [\varepsilon_t (b_{t-2} + g R_t^{-1} \hat{y}_{t-2} [(T_{b,t-2} - b_{t-2}) \hat{y}_{t-2} + V_{b,t-2} \varepsilon_{t-1}]) \qquad (19) \\ = \; & T^{-1} \sum_{t=1}^{T} (\varepsilon_t b_{t-2}) + g R_t^{-1} T^{-1} \sum_{t=1}^{T} (\varepsilon_t \hat{y}_{t-2} [(T_{b,t-2} - b_{t-2}) \hat{y}_{t-2} + V_{b,t-2} \varepsilon_t]) \end{aligned}$$

And $p \lim_{T \longrightarrow \infty} T^{-1} \sum_{t=1}^{T} (\varepsilon_t b_{t-1}) = 0$. Therefore, $p \lim_{T \longrightarrow \infty} T^{-1} \sum_{t=1}^{T} (X_t) = 0$. Similar reasoning as above implies $p \lim_{T \longrightarrow \infty} T^{-1} \sum_{t=1}^{T} (e_t) = 0$. For $T^{-1} \sum_{t=1}^{T} (X_t e_t)$:

$$T^{-1} \sum_{t=1}^{T} (X_t e_t) = T^{-1} \sum_{t=1}^{T} \left[\begin{array}{c} \frac{1}{2} T_{b,t-1} (1-\rho) \hat{y}_{t-1} \varepsilon_t (-T_{b,t} + \frac{1}{2} (V_{b,t-1} + T_{b,t-1})) \\ + \frac{1}{2} T_{b,t-1} (1-\rho) \hat{y}_{t-1} V_{b,t} \varepsilon_{t+1} \\ - \frac{1}{2} \varepsilon_t (T_{b,t-1} - V_{b,t-1}) \varepsilon_t (-T_{b,t} + \frac{1}{2} (V_{b,t-1} + T_{b,t-1})) \\ - \frac{1}{2} \varepsilon_t (T_{b,t-1} - V_{b,t-1}) V_{b,t} \varepsilon_{t+1} \end{array} \right] \tag{20}$$

Again, using the independence of b_t^2 and ε_t, $T^{-1} \sum_{t=1}^{T} (X_t e_t)$ reduces to:

$$T^{-1} \sum_{t=1}^{T} (X_t e_t) = -\frac{1}{2} T^{-1} \sum_{t=1}^{T} [\varepsilon_t^2 (T_{b,t-1} - V_{b,t-1}) (-T_{b,t} + \frac{1}{2} (V_{b,t-1} + T_{b,t-1}))] \tag{21}$$

Also,

$$\begin{aligned} & T^{-1} \sum_{t=1}^{T} [(T_{b,t-1} - V_{b,t-1}) T_{b,t}] \qquad (22) \\ = \; & (1-\rho) \left(\frac{\rho}{\sigma} \right)^2 - (1-\rho) \rho T^{-1} \sum_{t=1}^{T} \left(\frac{\beta b_{t-1}}{1-\beta\rho} \frac{\beta b_t}{1-\beta\rho} \right) \\ & -2 \frac{\rho^2 (1-\rho)^2 \beta\rho (1-\beta)}{\sigma^2 (1-\beta\rho)^2} - \frac{\rho^3 (1-\rho)(1-\beta)^2}{\sigma^2 (1-\beta\rho)^2} \end{aligned}$$

and as $t \longrightarrow \infty$ $E(b_t^2) = gC + \frac{\rho^2(1-\rho)^2}{\sigma^2}$,

$$T^{-1}\sum_{t=1}^{T}(T_{b,t}^2 - V_{b,t}^2) = \left[\frac{\beta^2(\rho^2-1)}{(1-\beta\rho)^2}\right]\left(gC + \rho^2(1-\rho^2)\right) \tag{23}$$
$$+2\frac{\rho(1-\rho)}{\sigma}\left[\frac{\beta(1-\rho)}{\sigma(1-\beta\rho)^2}(1-\rho^2)\right] - \frac{(1-\rho)^2}{\sigma^2(1-\beta\rho)^2}\left[1-\rho^2\right]$$

Finally, I have:

$$T^{-1}\sum_{t=1}^{T}(X_t e_t) = \frac{\sigma_\varepsilon^2}{2}\left[\begin{array}{c} (1-\rho)\left(\frac{\rho}{\sigma}\right)^2 - (1-\rho)\rho T^{-1}\sum_{t=1}^{T}\left(\frac{\beta b_{t-1}}{1-\beta\rho}\frac{\beta b_t}{1-\beta\rho}\right) \\ -2\frac{\rho^2(1-\rho)^2\beta\rho(1-\beta)}{\sigma^2(1-\beta\rho)^2} - \frac{\rho^3(1-\rho)(1-\beta)^2}{\sigma^2(1-\beta\rho)^2} \end{array}\right] \tag{24}$$
$$+\frac{\sigma_\varepsilon^2}{2}\left[\begin{array}{c} \left[\frac{\beta^2(\rho^2-1)}{(1-\beta\rho)^2}\right]\left(gC + \rho^2(1-\rho^2)\right) \\ +2\frac{\rho(1-\rho)}{\sigma}\left[\frac{\beta(1-\rho)}{\sigma(1-\beta\rho)^2}(1-\rho^2)\right] \\ -\frac{(1-\rho)^2}{\sigma^2(1-\beta\rho)^2}\left[1-\rho^2\right] \end{array}\right]$$

Using $E(b_t\hat{y}_t^2)$ and $E(b_t^2\hat{y}_t^2) = 0$, and $E(b_t b_{t+1}) = E(b_t^2)$, we get:

$$\frac{p\lim_{T\longrightarrow\infty} T^{-1}\sum_{t=1}^{T}(X_t e_t)}{p\lim_{T\longrightarrow\infty}(T^{-1}\sum_{t=1}^{T}X_t^2} - 1 = \frac{\sigma_\varepsilon^2}{2}\left[A - \frac{\beta^2(1-\rho)}{(1-\beta\rho)^2}E(b_t^2)\right] - 1 \tag{25}$$

where

$$A = (1-\rho)\left(\frac{\rho}{\sigma}\right)^2 - 2\frac{\rho^2(1-\rho)^2\beta\rho(1-\beta)}{\sigma^2(1-\beta\rho)^2} - \frac{\rho^3(1-\rho)(1-\beta)^2}{\sigma^2(1-\beta\rho)^2} \tag{26}$$
$$+2\frac{\rho(1-\rho)}{\sigma}\left[\frac{\beta(1-\rho)}{\sigma(1-\beta\rho)^2}(1-\rho^2)\right] - \frac{(1-\rho)^2}{\sigma^2(1-\beta\rho)^2}\left[1-\rho^2\right]$$

is negative. Thus, the bias is negative, and the slope estimator γ is less than one. For longer yields, the same reasoning applies, as the $T-$mappings are monotonic transformations of the mappings considered here.

A. 4: Proof of Proposition 3:

Under RE, the one-period asset price is $\hat{P}_{1,t}^{RE} = \frac{\rho(1-\rho)}{\sigma}\hat{y}_{t-1} + \frac{(1-\rho)}{\sigma}\varepsilon_t$ and its variance is $V(P_{1,t}^{RE}) = \frac{\rho^2(1-\rho)^2}{\sigma^2}E(\hat{y}_{t-1}^2) + \frac{(1-\rho)^2}{\sigma^2}E(\varepsilon_t^2)$. Under learning, the corresponding variables are $\hat{P}_{1,t}^{L} = T_a + T_b\hat{y}_{t-1} + V_b\varepsilon_t$, and $V(\hat{P}_{1,t}^{L}) = E\left[T_a + T_b\hat{y}_{t-1} + V_b\varepsilon_t - E(T_a + T_b\hat{y}_{t-1} + V_b\varepsilon_t)\right]^2$.

For evaluating the unconditional expectations, as $g \longrightarrow \bar{g}$, gt becomes large, $E(T_a) = 0$ and $E(T_b\hat{y}_{t-1}) = E\left[\left(\rho\left[\frac{-\beta b_{t-1}}{1-\beta\rho} - \frac{(1-\beta)\rho}{\sigma(1-\beta\rho)}\right] + \frac{\rho}{\sigma}\right)\hat{y}_{t-1}\right] = 0$. Similarly, $E(V_b\varepsilon_t) = 0$.

Now, the mappings are:

$$T_a = \frac{-\beta}{1-\beta}a_t;\ T_b = \rho\left[\frac{-\beta b_t}{1-\beta\rho} - \frac{(1-\beta)\rho}{\sigma(1-\beta\rho)}\right] + \frac{\rho}{\sigma} \tag{27}$$
$$V_b = \left[\frac{-\beta b_t}{1-\beta\rho} - \frac{(1-\beta)\rho}{\sigma(1-\beta\rho)}\right] + \frac{1}{\sigma}$$

Then the difference between learning and RE variance is given by:

$$V(\hat{P}_{1,t}^{L}) - V(\hat{P}_{1,t}^{RE}) = \frac{\beta^2}{(1-\beta)^2}E(a_t^2) \tag{28}$$
$$+E(\hat{y}_{t-1}^2)\left[\left(-\frac{(1-\beta)\rho^2}{\sigma(1-\beta\rho)} + \frac{\rho}{\sigma}\right)^2 - \frac{\rho^2(1-\rho)^2}{\sigma^2}\right]$$
$$+E(\varepsilon_t^2)\left[\left(-\frac{(1-\beta)\rho}{\sigma(1-\beta\rho)} + \frac{1}{\sigma}\right)^2 - \frac{(1-\rho)^2}{\sigma^2}\right]$$

The constant terms in second and third terms are positive for all values of σ and $\beta, \rho \in (0,1)$. Therefore, $V(\hat{P}_{1,t}^{L}) = V(\hat{P}_{1,t}^{RE}) + f(V(a_t)$,positive constants). As the one-period yield is a linear transformation of the one-period price, the result follows.

References

[1] Evans, George and Seppo Honkapohja. (2001). Learning and Expectations in Macroeconomics. Princeton University Press, Princeton, New Jersey.